Notes 15: Sec. 4-1 Graph Quadratic Equations in Standard Form and Sec. 4-2 Graph Quadratic Functions in Vertex or Intercept Form

A Quadratic function can be written in the standard form _______________.
Its graph is U shaped and is called a _______________.
Label the key parts of the graph:

- Quadratic functions open UP when _________________ & open DOWN when _______________.

Example 1
Graph a function of the form \( y = ax^2 + c \)
Graph \( y = -2x^2 + 2 \). Compare the graph with the graph of \( y = x^2 \).

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & \text{if} \ b=0 \ let \ x=-2, -1, 1, 2 \\
-2 & -6 & y=-2x^2+2 \\
-1 & 0 & y=-2(\ )^2+2 \\
0 & 2 & \\
1 & 0 & \\
2 & -6 & \\
\end{array}
\]

\( y = -2x^2 + 2 \) is opening down is narrower than \( y = x^2 \)
\( |a| \); \( |a| > 1 \) always narrower \( |a| < 1 \) wider
Example 2:
For the following functions (a) tell whether the graph opens up or opens down, (b) find the vertex, (c) find the axis of symmetry, and (d) graph it

\( y = -2x^2 - 1 \)

\( y = 3x^2 - 2 \)

**Properties of the Graph of** \( y = ax^2 + bx + c \)

Characteristics of the graph of \( y = ax^2 + bx + c \):

- The graph opens up if \( a > 0 \) and opens down if \( a < 0 \). \( \text{Negative} \ "x^2" \)
- The graph is narrower than the graph of \( y = x^2 \) if \( |a| > 1 \) and wider if \( |a| < 1 \).
- The axis of symmetry is \( x = \frac{-b}{2a} \) and the vertex has \( x \)-coordinate \( \frac{-b}{2a} \).
- The \( y \)-intercept is \( c \). So, the point \((0, c)\) is on the parabola.
Example 3
Graph a function of the form $y = ax^2 + bx + c$
Graph $y = -x^2 + 4x - 3$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-x^2 + 4x - 3$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 + 0 - 3</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-1 + 4 - 3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-4 + 8 - 3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-9 + 12 - 3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-16 + 16 - 3</td>
<td>-3</td>
</tr>
</tbody>
</table>

\[ \chi = \frac{-b}{2a} = \frac{-4}{-2(-1)} = 2 \]

\[ \chi = 2 \]

- opens down
- same width as $x^2$
- line of symmetry
- \[ \text{Vertex (2, 1)} \]

**MINIMUM AND MAXIMUM VALUES**

For $y = ax^2 + bx + c$, the vertex's $y$-coordinate is the **minimum value** of the function if $a \leq 0$ and the **maximum value** if $a \geq 0$.

**Example 4:** Tell whether the function $y = -3x^2 + 12x - 6$ has a minimum value or a maximum value. Then find the minimum or maximum value.

\[ a = -3 \quad \text{down} \]

Maximum @ $(2, 6)$

\[ \chi = \frac{-b}{2a} = \frac{-(12)}{2(-3)} = \frac{-12}{-6} = 2 \]

\[ y = -3(2)^2 + 12(2) - 6 \]

\[ y = -12 + 24 - 6 = 6 \]
Another useful form of the quadratic function is the vertex form: \[ y = a(x-h)^2 + k. \]

**GRAPH OF VERTEX FORM** \( y = a(x-h)^2 + k \)

The graph of \( y = a(x-h)^2 + k \) is the parabola \( y = ax^2 \) translated \( h \) units and \( k \) units.
- The vertex is \((h, k)\).
- The axis of symmetry is \( x = \frac{h}{2} \).
- The graph opens up if \( a > 0 \) and down if \( a < 0 \).

**Example 5**
Graph a quadratic function in vertex form

Graph \( y = (x+1)^2 - 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>((x+1)^2 - 2)</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>((-2)^2 - 2)</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>((-1)^2 - 2)</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>((0)^2 - 2)</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>((1)^2 - 2)</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>((2)^2 - 2)</td>
<td>2</td>
</tr>
</tbody>
</table>

**Example 6:** Write a quadratic function in vertex form for the function whose graph has its vertex at \((-5, 4)\) and passes through the point \((7, 1)\).
GRAPH OF INTERCEPT FORM \( y = a(x - p)(x - q) \):

Characteristics of the graph \( y = a(x - p)(x - q) \):
- The \( x \)-intercepts are \( p \) and \( q \).
- The axis of symmetry is halfway between \((p, 0)\) and \((q, 0)\) and it has equation \( x = \frac{p + q}{2} \).
- The graph opens up if \( a \geq 0 \) and opens down if \( a \leq 0 \).

Example 7:
Graph \( y = -2(x - 1)(x - 5) \).

\( \chi = 1 \quad (1, 0) \)
\( \chi = 5 \quad (5, 0) \)

**\( \chi \)-value of vertex**

\[ \chi = \frac{1 + 5}{2} = \frac{6}{2} = 3 \]

\( y = -2(3-1)(3-5) \)
\( y = -2(2)(-2) = 8 \)

To change from intercept form to Standard form: **Ex:** \( y = -2(x + 5)(x - 8) \)

1. Use foil to multiply binomials together: \( y = -2(x^2 - 3x - 40) \)
2. Distribute coefficient to quadratic: \( y = -2x^2 + 6x + 80 \)
To change from vertex form to standard form:  

**Ex:** \( f(x) = 4(x-1)^2 + 9 \)

1. Foil binomial: 
   \[
f(x) = 4(x-1)(x-1) + 9 \rightarrow 4(x^2 - 2x - x + 1) + 9 \]
   \[
f(x) = 4(x^2 - 3x + 1) + 9 \]

2. Distribute coefficient to quadratic: 
   \[
f(x) = 4x^2 - 12x + 4 + 9 \]
   \[
f(x) = 4x^2 - 12x + 13 \]

3. Combine like terms: 
   \[
f(x) = 4x^2 - 12x + 13 \]

**HW #15:** pg. 240 #3-39 by 3’s and pg. 249 #4-40 by 4’s
Please use graph paper on this assignment