MATHEMATICS : THE LANGUAGE OF PHYSICS

1. QUADRATIC EQUATIONS : ALWAYS HAVE TWO SOLUTIONS.

a) FACTORIZATION: $x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0 \quad \therefore \quad x = 8 \qquad x = -3.$

b) QUADRATIC FORMULA: $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$2x^2 + x - 28 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - (4)(2)(-28)}}{2(2)} = \frac{-1 \pm \sqrt{1 + 224}}{4}$$

$$x = (-1 \pm 15)/4 = \begin{cases} 3.5 \\ -4 \end{cases}$$

2. EXPONENTIAL NOTATION : EXAMPLE:

$x = 4000 = 4 \times 10^3 \qquad y = .05 = 5 \times 10^{-2}$

a) $xy = (4 \times 10^3)(5 \times 10^{-2}) = 20 \times 1^1 = 2 \times 10^2$

b) $x/y = (4/5) \times 10^{3-(-2)} = .8 \times 10^5 = 8 \times 10^4$

ON A CALCULATOR: TO MULTIPLY:

4, EXP, 3, X, 5, EXP, 2, +/-, =

3. SIMULTANEOUS EQUATIONS: REQUIRE AS MANY EQUATIONS AS THE NUMBER OF UNKNOWNS.

EXAMPLE:

$$\begin{cases} 3x - y = 10 \\ 5x + 7y = 34 \end{cases}$$

$$\begin{cases} 21x - 7y = 70 \\ 5x + 7y = 34 \end{cases}$$

$26x = 104$

$x = 4$

$\therefore \quad y = 2$

EXAMPLE:

$$\begin{cases} 5A = 50 - T_2 \\ 2A = T_2 - T_1 \\ 3A = T_1 - 30 \end{cases}$$

$10A = 20$

$\therefore \quad A = 2$

SUBSTITUTE ABOVE:

$10 = 50 - T_2 \quad \therefore T_2 = 40$

$6 = T_1 - 30 \quad \therefore T_1 = 36$

4. LOGARITHMS LOG BASE 10

KEY: LOG X AND 10^x ARE OPPOSITES.

EXAMPLE: GIVEN: $10^x = 25$. FIND X.

$x = \log 25$

$x = 1.4$

EXAMPLE: GIVEN: $\log X = 1.8$ FIND X.

$$X = 10^{1.8} = 63$$

ON A CALCULATOR: METHOD 1: 10, y^x, 1.8, =

METHOD 2: 1.8, INV, LOG

5. NATURAL LOGARITHMS LN BASE e

WHERE $e = 2.7182818...$ $\ln X$ AND e^X ARE OPPOSITES.

EXAMPLE: GIVEN: $e^X = 4$

$$X = \ln 4$$
$$X = 1.386$$

EXAMPLE: GIVEN: $\ln X = 7$

$$X = e^7$$
$$X = 1096.6$$

6. LOGARITHM RULES: FOR ANY BASE:

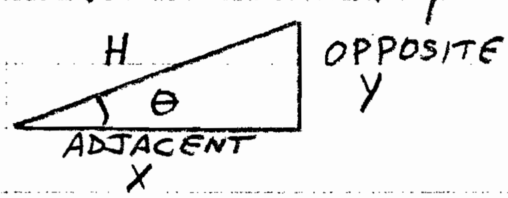
a) $\log A + \log B = \log(AB)$

b) $\log A - \log B = \log(A/B)$

c) $N \log A = \log(A^N)$

d) $\log_A B = (\log_N B)/(\log_N A)$

7. TRIGONOMETRY: RIGHT TRIANGLES



$$X^2 + Y^2 = H^2$$
$$\cos^2\theta + \sin^2\theta = 1$$

$\cos\theta = X/H$ $\sin\theta = Y/H$

$X = H\cos\theta$ $Y = H\sin\theta$

NOTE: $\sin\theta$ AND $\cos\theta$ ARE FRACTIONS AND ARE THEREFORE DILUTION FACTORS.

$\tan\theta = Y/X$ $\therefore$ $\theta = \tan^{-1}(Y/X)$

NOTE: TAN AND $\tan^{-1}$ ARE INVERSE FUNCTIONS.

$\tan\theta = Y/X$ MEANS THAT WE KNOW THE ANGLE AND WE WANT TO FIND Y OR X.

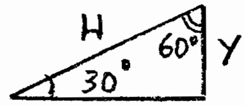
$\theta = \tan^{-1}(Y/X)$ MEANS THAT WE KNOW X AND Y AND WE WISH TO FIND θ.

EXAMPLE: GIVEN: $\sin\theta = .866$ FIND θ.

$$\theta = \sin^{-1}(.866) = 60^\circ$$

THE FUNCTION OF AN ANGLE EQUALS THE COFUNCTION OF THE COMPLEMENTARY ANGLE. $\sin 30^0 = \cos 60^0$

JUSTIFICATION:

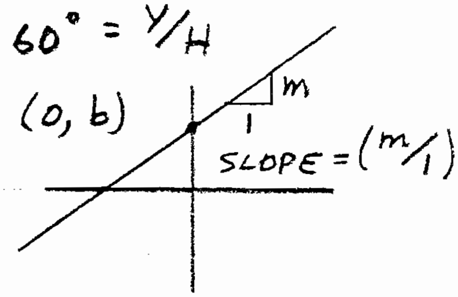


$\sin 30^\circ = Y/H$

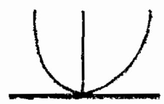
$\cos 60^\circ = Y/H$

8. GRAPHING:

a) LINEAR: $y = mx + b$



b) PARABOLIC: $y = ax^2$



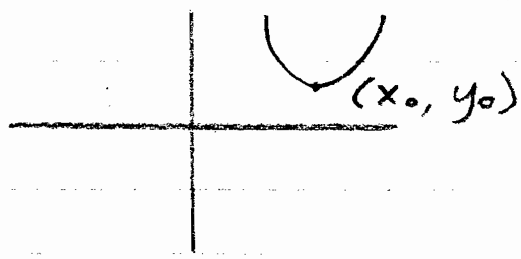
IF $a = (+)$, THE PARABOLA HOLDS WATER.

IF $a = (-)$, THE PARABOLA OPENS DOWNWARD.

IF a IS A LARGE NUMBER, THE PARABOLA IS SKINNY.

IF THE PARABOLA IS NOT CENTERED AT THE ORIGIN:

$(y - y_0) = a(x - x_0)^2$



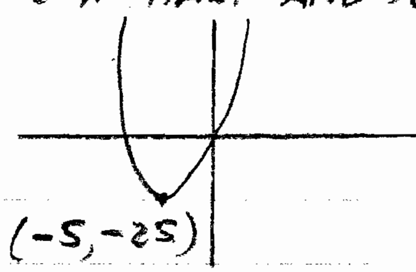
EXAMPLE: GIVEN:

$y = x^2 + 10x$ SKETCH THIS PARABOLA.

COMPLETE THE SQUARE: DIVIDE IN HALF AND SQUARE.

$y + 25 = x^2 + 10x + 25$

$(y + 25) = (x + 5)^2$

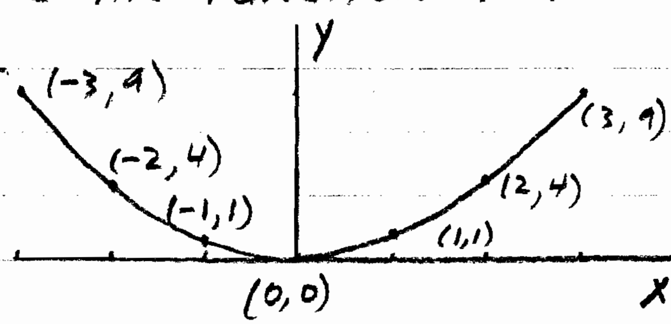


9. DERIVATIVES: PURPOSE: TO FIND THE SLOPE OF A CURVE. EXAMPLE: GIVEN: $y = x^2$

TO PLOT THIS CURVE, USE THE FUNCTION $y = x^2$

X	Y
0	0
1	1
2	4
3	9

X	Y
−1	1
−2	4
−3	9

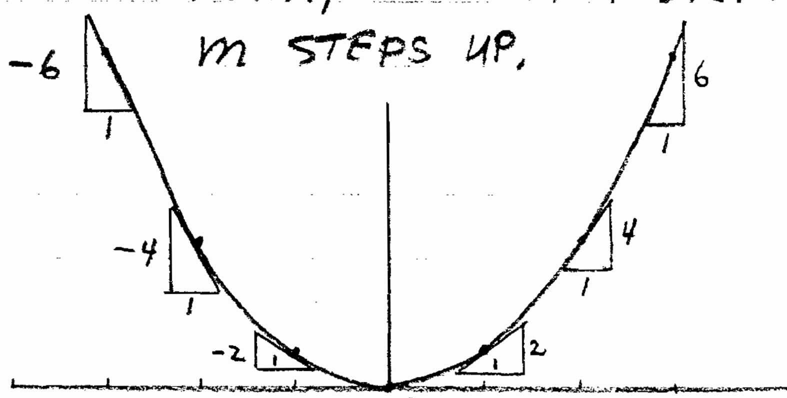


TO FIND THE STEEPNESS OF THIS CURVE, THAT IS, ITS SLOPE, WE TAKE THE DERIVATIVE. FOR POLYNOMIALS, BRING DOWN THE POWER AND MAKE IT ONE LESS POWER. FOR $y = x^2$:

$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{\text{CHANGE IN } Y}{\text{CHANGE IN } X} = \frac{dy}{dx} = 2x$$

X	dy/dx = SLOPE = $2X$
0	0
1	2
2	4
3	6
−1	−2
−2	−4
−3	−6

NOTE: THE SLOPE IS ALWAYS ONE STEP OVER, m STEPS UP.



dy = SMALL PIECE OF y dx = SMALL PIECE OF X.

EXAMPLE: GIVEN: $Y = 7X^4 + 2X^3 + X^1 + 7X^0 + X^{-1}$

FIND A FORMULA FOR THE SLOPE.

$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{dy}{dx} = 28X^3 + 6X^2 + 1 + 0 - 1X^{-2}$$

FIND OUR LOCATION AT $X = 2$.

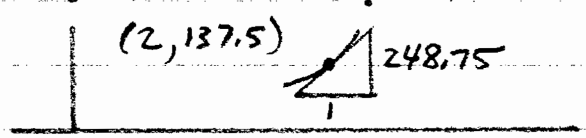
$$Y = 7(2)^4 + 2(2)^3 + 2 + 7 + \tfrac{1}{2} = 137.5$$

$$(X, y) = (2, 137.5)$$

FIND THE SLOPE AT $X = 2$.

$$dy/dx = 28(2)^3 + 6(2)^2 + 1 - \tfrac{1}{2^2} = 248.75 / 1$$

SKETCH:



10. INTEGRALS: OPPOSITE OF DERIVATIVES

PURPOSE: TO FIND AN EQUATION FOR y IF WE KNOW THE SLOPE EQUATION.

EXAMPLE: GIVEN: $dy/dx = 2X$.

FIND AN EQUATION FOR y.

$$dy/dx = 2x$$

$$dy = (2x)\,dx$$

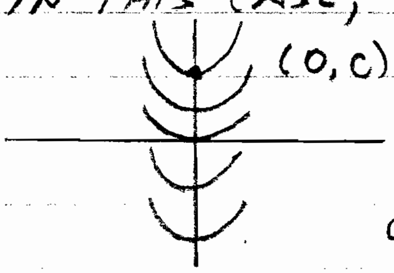
$$\int dy = \int (2x)\,dx$$

$$y = 2\,\frac{x^2}{2} + C \qquad \therefore\ y = x^2 + C$$

NOTE: $\int$ MEANS "ADD THEM ALL UP"
$\therefore \int dy$ MEANS "ADD UP ALL THE SMALL PIECES OF y."

AN INTEGRAL SYMBOL MUST ALWAYS BE ACCOMPANIED BY A DIFFERENTIAL AT THE END. THE QUANTITY CONTAINED IN THE INTERIOR PARENTHESES MAY ON BE CONSTANTS OR THE SAME VARIABLE AS THE FINAL DIFFERENTIAL. RECIPE: THE $\int$ AND THE DIFFERENTIAL ACT ON THE INTERIOR FUNCTION AND THEN BOTH DISAPPEAR, FOR POLYNOMIALS, RAISE X TO ONE HIGHER POWER AND DIVIDE BY THE NEW POWER.

THE REASON WE INCLUDE " + C " WHERE C IS A CONSTANT IS THAT WE WERE GIVEN THE SHAPE OF THE CURVE, BUT NOT ITS LOCATION. IN THIS CASE, C IS THE y-INTERCEPT.



EXAMPLE: GIVEN:

$$dy/dx = x^3 + 6x^2 + x + 2.$$

FIND AN EQUATION FOR y. $\quad dy = (x^3 + 6x^2 + x + 2)\,dx$

$$\int dy = \int (x^3 + 6x^2 + x + 2)\,dx$$

$$y = \frac{x^4}{4} + \frac{6x^3}{3} + \frac{x^2}{2} + \frac{2x}{1} + C$$

UNITS AND CHAIN CALCULATIONS

I. METRIC SYSTEM : SI = SYSTEM INTERNATIONAL

A. BASIC UNITS :

LENGTH [METERS] — WAVELENGTHS OF LIGHT FROM KRYPTON - 86.

TIME [SECONDS] — PERIODS OF LIGHT FROM CESIUM - 133.

MASS [KILOGRAMS] — Pt - Ir ALLOY CYLINDER IN PARIS, FRANCE.

B. PREFIXES :

LARGE

K = KILO = THOUSAND

M = MEGA = MILLION

SMALL

c = CENTI = ONE - HUNDREDTH

m = milli = ONE - THOUSANDTH

μ = MICRO = ONE - MILLIONTH

EXAMPLES :

1 km = 1000 m

100 cm = 1 m

1000 m# = 1 #

$1 \times 10^6 \mu g = 1 g$

$1 Ms = 1 \times 10^6$ SECONDS

C. UNIT CONVERSIONS

KEY : WE WILL MULTIPLY BY FACTORS EQUAL TO ONE. ∴ THE NET VALUE WILL NOT CHANGE. IN EACH CONVERSION FACTOR, THE DIVISION SIGN CAN BE READ AS AN EQUAL SIGN.

EXAMPLE :

$$50 \text{ Mg} \left(\frac{1\times10^6 \text{ g}}{1 \text{ Mg}}\right)\left(\frac{1\times10^3 \text{ mg}}{1 \text{ g}}\right) = 5\times10^{10} \text{ mg}$$

$$45 \text{ cm}\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right) = 450 \text{ mm}$$

$$9700 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ INCH}}{2.54 \text{ cm}}\right)\left(\frac{1 \text{ ft}}{12''}\right)\left(\frac{1 \text{ mile}}{5280 \text{ ft}}\right) = 6 \text{ miles}$$

$$\frac{55 \text{ m}}{\text{SEC}}\left(\frac{1 \text{ mile}}{1609.3 \text{ m}}\right)\left(\frac{60 \text{ SEC}}{1 \text{ MIN}}\right)\left(\frac{60 \text{ MIN}}{1 \text{ HOUR}}\right) = 123 \frac{\text{miles}}{\text{HR}}$$

$$18 \text{ cm}^2\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2\left(\frac{1 \text{ Km}}{1000 \text{ m}}\right)^2 = 1.8\times10^{-9} \text{ Km}^2$$

$$5\ \frac{m^2}{HR^3}\left(\frac{100\ cm}{1m}\right)^2\left(\frac{1\ HR}{60\ MIN}\right)^3\left(\frac{1\ MIN}{60\ SEC}\right)^3 = 1.07\times10^{-6}\ \frac{cm^2}{SEC^3}$$

D. PRACTICAL APPLICATIONS

RECIPE: 1. IDENTIFY CONVERSION FACTORS: MIXED UNITS, g/cm^3, m/s, ...; PERCENT; PER.

2. WRITE THE NAME OF THE UNKNOWN =

3. START THE CHAIN WITH THE REMAINING NUMBER.

EXAMPLE: JESSE'S FERRARI ZOOMS AT 40 m/s FOR 80 SEC. FIND THE DISTANCE TRAVELED.

$$x = 80\ sec\ (40\ m/s) = 3200\ m$$

EXAMPLE: MICHAEL'S BMW CRUISES AT 30 m/s. FIND THE TIME TO TRAVEL 5250 m.

$$t = 5250\ m\left(\frac{1\ SEC}{30\ m}\right) = 175\ \text{SECONDS}$$

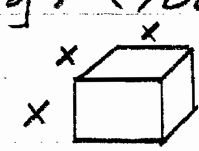
EXAMPLE: DIAMOND HAS A DENSITY OF $3.5\ g/cm^3$. FIND THE MASS, IN mg, OF A JEWEL WHOSE VOLUME IS $60\ mm^3$.

$$\text{MASS} = 60\ mm^3\left(\frac{1\ cm}{10\ mm}\right)^3\left(\frac{3.5\ g}{1\ cm^3}\right)\left(\frac{1000\ mg}{1\ g}\right) = 210\ mg$$

EXAMPLE: FIND THE VOLUME OF A 21600 KG CUBE OF ALUMINUM WHOSE DENSITY IS $2.7\ g/cm^3$.

$$V = 21600\ kg\left(\frac{1000\ g}{1\ kg}\right)\left(\frac{1\ cm^3}{2.7\ g}\right)\left(\frac{1\ m}{100\ cm}\right)^3 = 8\ m^3$$

FIND THE LENGTH OF ONE SIDE OF THIS CUBE.



$$x^3 = 8\ m^3$$
$$x = \sqrt[3]{8} = 2\ m$$

EXAMPLE: A ROLL OF QUARTERS, WHICH CONTAINS 40 COINS, HAS A TOTAL MASS OF 350 g. A QUARTER IS 70% COPPER. FIND THE MASS OF COPPER IN \$50 OF COINS.

$$\text{MASS OF COPPER} = \$50\left(\frac{4\ \text{COINS}}{\$1}\right)\left(\frac{1\ \text{ROLL}}{40\ \text{COINS}}\right)\left(\frac{350\ g}{1\ \text{ROLL}}\right)\left(\frac{70\ g\ Cu}{100\ g\ \text{COINS}}\right)$$

MASS Cu = 1225 g

NOTE : 1) CARAT = 200 mg   2) KARAT = $\frac{1}{24}$ th

∴ 18 K GOLD IS $18/24$ th GOLD.