STATICS = STUDY OF OBJECTS WHOSE $\vec{a} = 0$.

I. TYPES OF EQUILIBRIUM

A. EQUILIBRIUM OCCURS IF $\vec{a}$ OF THE OBJECT IS ZERO.

B. STATIC EQUILIBRIUM : BOTH $\vec{a} = 0$ AND $\vec{V} = 0$.

C. DYNAMIC EQUILIBRIUM : $\vec{a} = 0$ BUT $\vec{V}$ = SOME CONSTANT. THUS, THE MASS MOVES AT CONSTANT SPEED IN A STRAIGHT LINE.

II. EQUILIBRIUM OF A PARTICLE

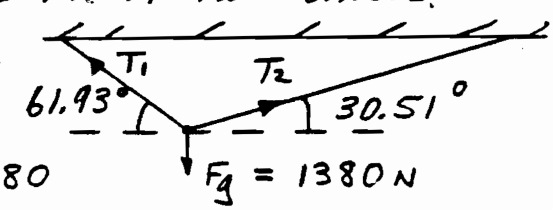
A. PARTICLE = A POINT MASS WITH ALL FORCES CONCENTRATED ON THAT POINT.

B. RECIPE :

1. PUT A DOT ON THE CENTER OF THE OBJECT.
2. DRAW THE FORCES EMANATING FROM THE DOT.
3. WRITE THE X AND Y EQUATIONS FOR NEWTON'S SECOND LAW. $ma = \Sigma F$
4. SET $A_x = 0$ AND $A_y = 0$ AND SOLVE FOR THE UNKNOWNS.

C. EXAMPLES :

1. A 138 KG STREET LIGHT HANGS FROM TWO WIRES. FIND THE TENSION IN EACH ROPE.



$$\begin{cases} mA_x = T_2 \cos 30.51 - T_1 \cos 61.93 \\ mA_y = T_2 \sin 30.51 + T_1 \sin 61.93 - 1380 \end{cases}$$

$$\begin{cases} 0 = T_2 \cos 30.51 - T_1 \cos 61.93 \\ 0 = T_2 \sin 30.51 + T_1 \sin 61.93 - 1380 \end{cases}$$

$$\begin{cases} 0 = .8615\, T_2 - .4705\, T_1 \\ 0 = .5077\, T_2 + .8824\, T_1 - 1380 \end{cases}$$

$$\begin{cases} 0 = 1.831\, T_2 - T_1 \\ 0 = .5754\, T_2 + T_1 - 1564 \end{cases}$$

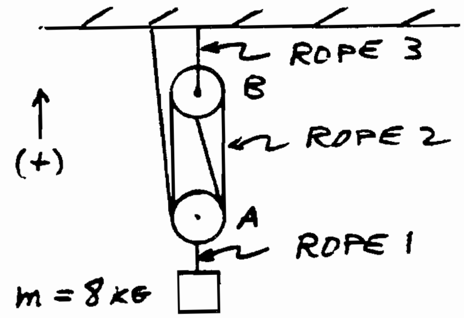
$$0 = 2.4064\, T_2 - 1564$$

$$\boxed{T_2 = 650\text{N}}$$

$$\therefore \boxed{T_1 = 1190\text{N}}$$

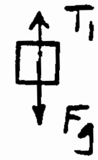
INTUITIONS: a) $T_1 > T_2$ BECAUSE ROPE 1 SUPPORTS MOST OF THE WEIGHT, ROPE 2 IS JUST HELPING TO PULL IT SIDEWAYS A BIT. b) TO PULL THE ROPES PERFECTLY HORIZONTAL, BOTH $T_1 = \infty$ AND $T_2 = \infty$.

2.



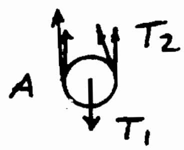
FIND THE TENSION IN EACH ROPE.

ROPE 1:



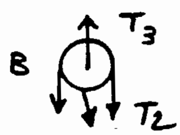
$$0 = T_1 - F_g$$
$$0 = T_1 - 80$$
$$\boxed{T_1 = 80N}$$

ROPE 2:



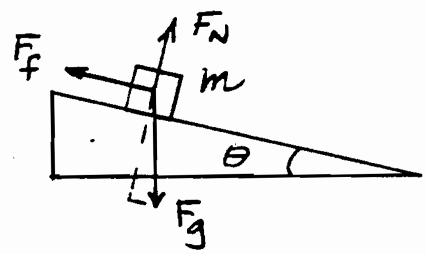
$$0 = 4T_2 - T_1$$
$$0 = 4T_2 - 80$$
$$\therefore \boxed{T_2 = 20N}$$

ROPE 3:



$$0 = T_3 - 3T_2$$
$$0 = T_3 - 60$$
$$\therefore \boxed{T_3 = 60N}$$

3. FIND A FORMULA FOR THE MINIMUM VALUE FOR μ SO THAT THE BRICK WILL NOT MOVE.



$$mA_x = F_g \sin\theta - F_f \qquad mA_y = F_N - F_g \cos\theta$$
$$0 = F_g \sin\theta - F_f \qquad 0 = F_N - F_g \cos\theta$$
$$F_f = F_g \sin\theta \qquad F_N = F_g \cos\theta$$

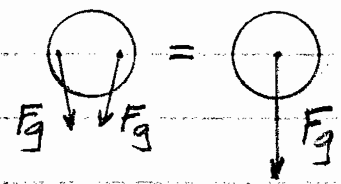
LEMMA: $F_f = \mu F_N$

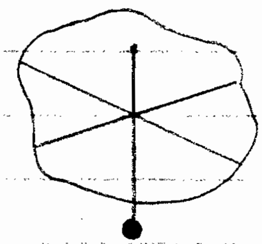
$$(F_g \sin\theta) = \mu (F_g \cos\theta)$$
$$\sin\theta = \mu \cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \mu$$
$$\therefore \boxed{\mu = \tan\theta}$$

III. EQUILIBRIUM OF A SOLID OBJECT

A. DEFINITIONS

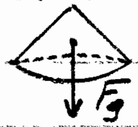
1. CENTER OF GRAVITY = THE POINT AT WHICH WE CAN CONCENTRATE ALL OF THE INDIVIDUAL GRAVITY VECTORS WITH ONE TOTAL F_g WHILE OBSERVING IDENTICAL RESULTS. INTUITIVELY, THE CENTER OF GRAVITY IS THE BALANCING POINT, WHICH DOES NOT NECESSARILY HAVE TO BE ON THE OBJECT. TO EXPERIMENTALLY FIND THE CENTER OF GRAVITY, WE USE A PLUMB BOB.



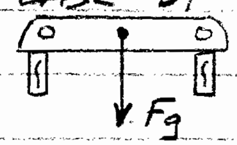


2. STABLE EQUILIBRIUM = AN OBJECT IS STATIONARY. IF WE DISTURB IT A BIT, THE OBJECT WILL RETURN TO ITS INITIAL POSITION. AN OBJECT WILL BE IN STABLE EQUILIBRIUM IF:

a) F_g VECTOR PIERCES THE BASE OF THE OBJECT.

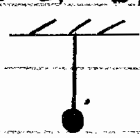


PARKED CAR

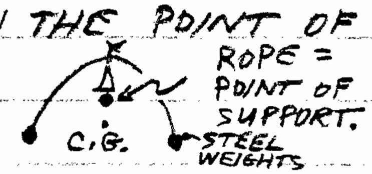


b) THE CENTER OF GRAVITY IS BELOW THE POINT OF SUPPORT.

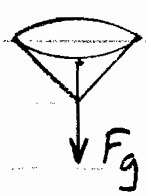
MASS HANGING ON A STRING



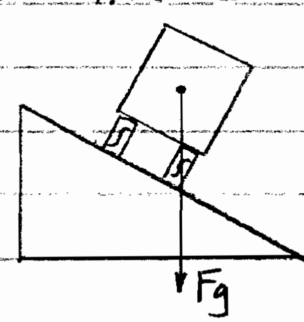
TIGHT ROPE WALKER



3. UNSTABLE EQUILIBRIUM = THE OBJECT IS STATIONARY. IF WE DISTURB THE OBJECT, IT WILL CONTINUE TO MOVE SO THAT IT LOWERS THE ELEVATION OF ITS CENTER OF GRAVITY.

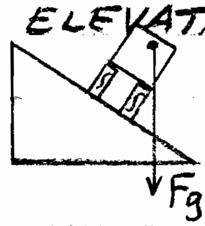


CAR PARKED ON A HILL

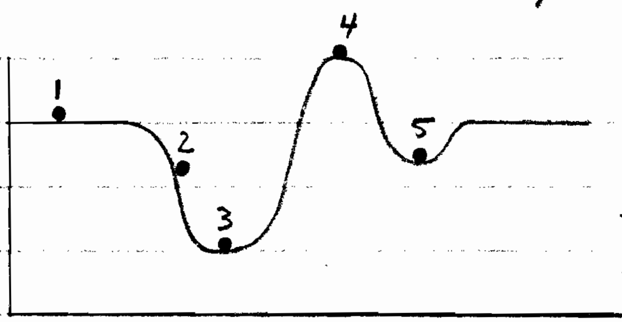


4. NEUTRAL EQUILIBRIUM = THE OBJECT IS STATIONARY. IF WE DISTURB IT, THE OBJECT WILL MOVE BUT THE CENTER OF GRAVITY REMAINS AT THE SAME ELEVATION.

5. OBJECTS NOT IN EQUILIBRIUM WILL SPONTANEOUSLY MOVE AS THEY LOWER THE ELEVATION OF THEIR CENTER OF GRAVITY.



PARKED TRUCK WITH TOP HEAVY LOAD.



1 = NEUTRAL EQUILIBRIUM
2 = NOT IN EQUILIBRIUM
3 = VERY STABLE EQUILIBRIUM
4 = UNSTABLE EQUILIBRIUM
5 = FAIRLY STABLE EQUILIBRIUM

B. TORQUE = ROTATIONAL ANALOGUE OF FORCE. FORCES CAUSE ACCELERATIONS. TORQUES CAUSE ANGULAR ACCELERATIONS, THAT IS, ROTATIONS.

$ma = \Sigma F \qquad I\alpha = \Sigma \tau$

I = MOMENT OF INERTIA [kg m^2]

α = ANGULAR ACCELERATION [sec^{-2}]

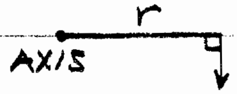
τ = TAU = TORQUE = [N m]

IN STATICS, WE WANT ON ROTATIONS. $\therefore \alpha = 0$.

$0 = \Sigma \tau$. 1. TO FIND TORQUE:

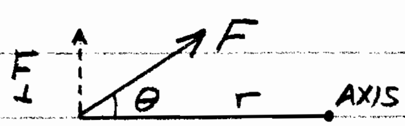
$\tau = r F \sin\theta \qquad \theta$ = ANGLE BETWEEN $\vec{r}$ AND $\vec{F}$.

EXAMPLE : LOOKING DOWN ON THE DOOR.

a) 

$\tau = rF\sin 90° = rF$

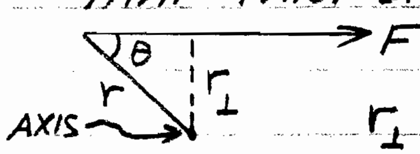
MOST EFFICIENT

b) 

$\tau = rF\sin\theta = r(F\sin\theta)$

$\tau = r F_\perp$

$F_\perp$ = THAT PART OF $\vec{F}$ WHICH CAUSES ROTATION.

c) 

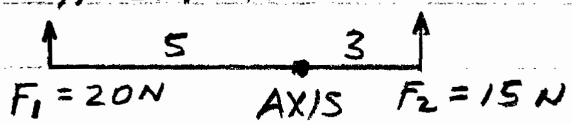
$\tau = (r\sin\theta)F = r_\perp F$

$r_\perp$ = SHORTEST DISTANCE FROM THE AXIS TO WHERE THE FORCE GOES BY.

2. UNITS: $\tau = [N\,m]$ OR $[ft\ lbs]$

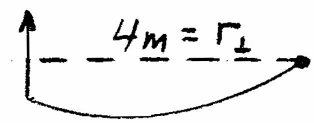
3. SIGNS: (+) CLOCKWISE, (−) COUNTERCLOCKWISE

4. EXAMPLE:



$\tau_1 = 100$ $\tau_2 = -45$

EXAMPLE:



$\tau = 200$

C. CONDITIONS FOR EQUILIBRIUM

$\vec{a} = 0$ $\alpha = 0$

$0 = \Sigma F_x$ RIGHT (+) LEFT (−) $0 = \Sigma F_y$ UP(+) DOWN(−) $0 = \Sigma \tau$ CW(+) CCW(−)

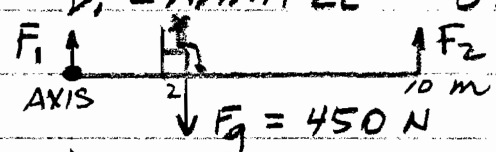
NO HORIZONTAL MOTION NO VERTICAL NO ROTATIONS

RECIPE: 1) DRAW A FORCE DIAGRAM FOR THE OBJECT.

2) PUT THE AXIS ON AN UNKNOWN FORCE.

3) WRITE THE THREE EQUILIBRIUM EQUATIONS, SOLVE.

D. EXAMPLE: GIVEN: ALL DISTANCES ARE MEASURED FROM THE AXIS, FIND F_1 AND F_2.



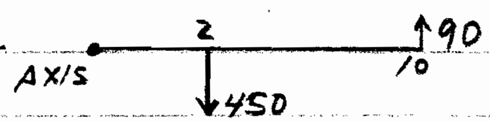
Y: $0 = F_1 + F_2 - 450$ | τ: $0 = (2)(450) - (10)(F_2)$

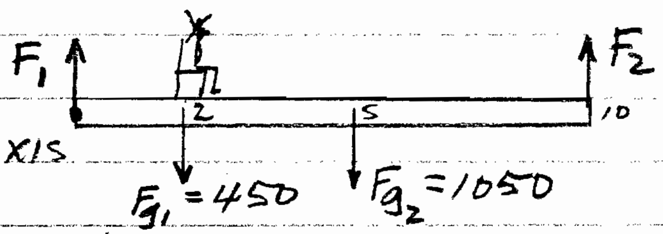
#2 $F_1 = 450 - 90 = 360$ #1 $F_2 = 90$

TO CHECK OUR ANSWER:

1. TOTAL FORCE UPWARD: $F_1 + F_2 = 450 = F_g$ ✓

2. $F_1 > F_2$ SINCE F_1 IS NEARER THE LOAD. ✓

3.  TORQUES BALANCE. ✓



EXAMPLE: GIVEN: AXIS

FIND F_1 AND F_2.

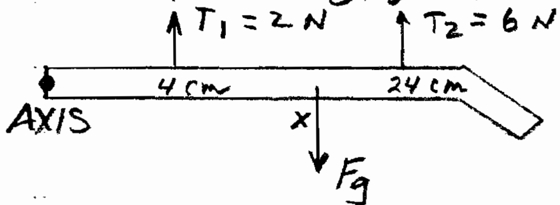
Y: $0 = F_1 + F_2 - 450 - 1050$ | τ: $0 = (2)(450) + (5)(1050) - 10F_2$

$F_1 = 885$ $F_2 = 615$

#2 #1

NOTE: PUT THE UNKNOWN FORCE AS UP. IF THE MATH GIVES A (−) NUMBER, THE FORCE IS ACTUALLY DOWN.

EXAMPLE: GIVEN:



FIND THE WEIGHT OF THE HOCKEY STICK AND THE LOCATION OF ITS CENTER OF GRAVITY.

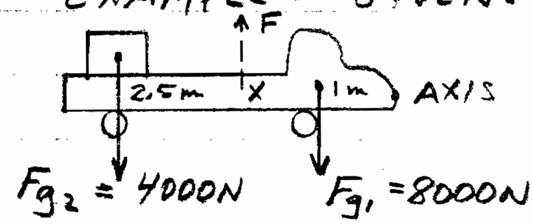
#1 y: $0 = 2 + 6 - F_g$

$F_g = 8N$

#2 τ: $0 = -(4)(2) - (24)(6) + x(8)$

$x = 19$ cm

EXAMPLE: GIVEN:



FIND THE LOCATION OF THE CENTER OF GRAVITY OF THE LOADED TRUCK. THAT IS, FIND THE LOCATION OF THE BALANCING FORCE, F.

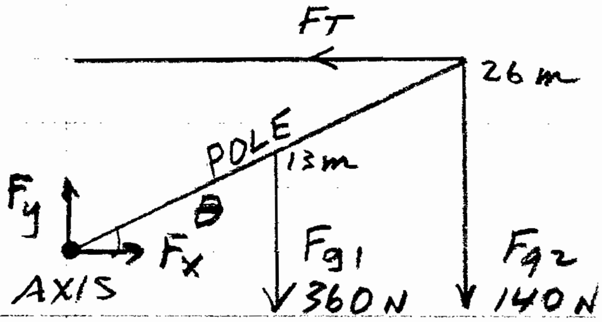
#1 y: $0 = F - 4000 - 8000$

$F = 12000N$

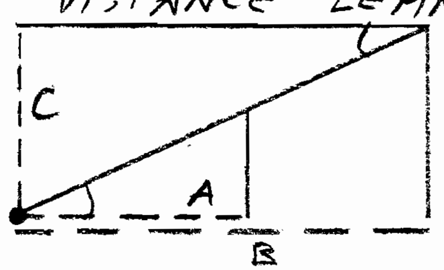
#2 τ: $0 = -(1)(8000) - (2.5)(4000) + x(12000)$

$x = 1.5$ m

EXAMPLE: A HORIZONTAL ROPE HOLD A POLE SLANTED AT AN ANGLE OF 22.6°. GIVEN:



DISTANCE LEMMAS:



$A = 13 \cos 22.6 = 12$

$B = 26 \cos 22.6 = 24$

$C = 26 \sin 22.6 = 10$

ONLY PURE VECTORS.

FIND THE FORCES EXERTED ON THE POLE.

#3 x: $0 = F_x - T$

$F_x = 768$

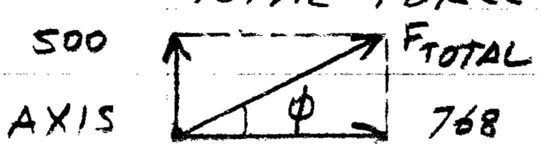
#2 y: $0 = F_y - 360 - 140$

$F_y = 500$

#1 τ: $0 = (12)(360) + (24)(140) - 10T$

$T = 768$

TOTAL FORCE EXERTED ON THE POLE BY THE WALL.

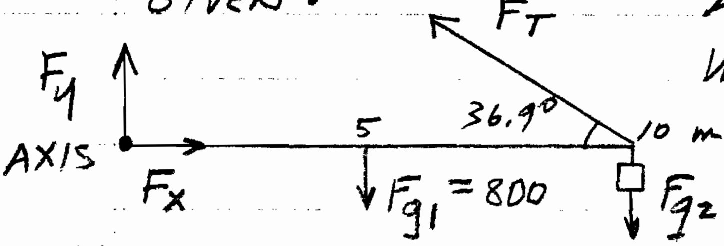


$F_{TOTAL} = \sqrt{F_x^2 + F_y^2} = 916$

$\phi = \tan^{-1} |F_y / F_x| = 33°$

EXAMPLE: MIXED VECTORS. A HORIZONTAL POLE IS SUPPORTED BY A WIRE WHOSE ANGLE IS 36.9°.

GIVEN:



ALL DISTANCES ARE MEASURED FROM THE AXIS. FIND THE FORCES EXERTED ON THE POLE.

X

$$0 = F_x - F_T \cos 36.9$$
$$F_x = 1000 \cos 36.9$$
$$F_x = 800$$

#2

Y

$$0 = F_y + F_T \sin 36.9 - 800 - 200$$
$$F_y = 800 + 200 - 1000 \sin 36.9$$
$$F_y = 400$$

#3

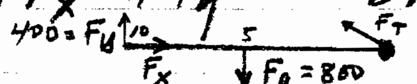
$$\tau: \quad 0 = (5)(800) + (10)(200) - (10)(F_T \sin 36.9°)$$
$$F_T = 1000$$

#1

TOTAL FORCE SUPPLIED AT AXIS

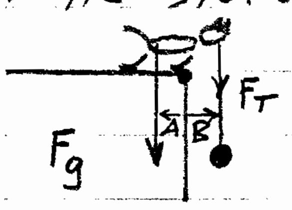
$$F_{TOTAL} = \sqrt{F_x^2 + F_y^2} = 894 \qquad \phi = \tan^{-1}|F_y/F_x| = 27°$$

TO CHECK, MOVE AXIS TO OTHER END.



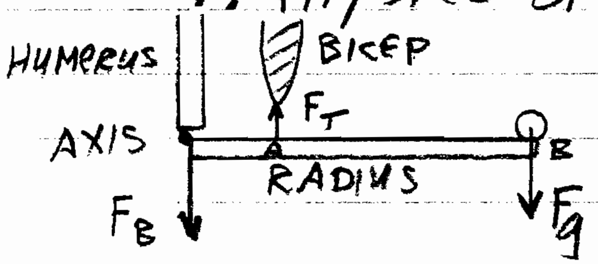
$$0 = -(5)(800) + (10)(400) \checkmark$$

E. PINK PANTHER: UNSTABLE FOR X-FORCES. ∴ HE MARCHES FORWARD. STABLE FOR Y-FORCES. ∴ HE STOPS AT EDGE.



$$\tau_{\text{CLOCKWISE}} = (B)(F_T)$$
$$\tau_{\text{COUNTERCLOCKWISE}} = (A)(F_g)$$
$$\tau_{cw} < \tau_{ccw} \quad \therefore \text{ HE IS SAFE.}$$

F. PHYSICS OF MUSCLES. GIVEN: FIND F_T AND F_{BONE}:

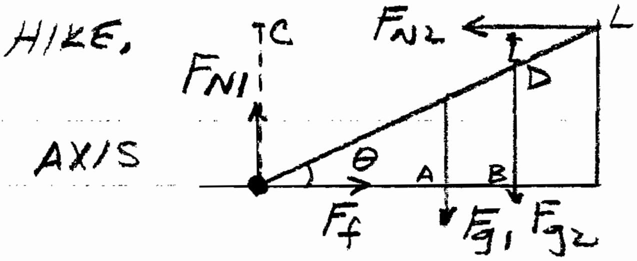


$$Y: \quad 0 = F_T - F_B - F_g$$
$$\tau: \quad 0 = -(A)(F_T) + (B)(F_g)$$

NOTE: A AND B ARE MEASURED FROM AXIS.

G. LADDERS: PUT THE AXIS AT FOOT OF LADDER.

EXAMPLE: GIVEN: LADDER: m_1, L, μ AND θ. PAINTER: m_2. FIND THE DISTANCE TO WHICH WE CAN SAFELY HIKE.



DISTANCE LEMMAS: $A = \frac{L}{2}\cos\theta$

$B = D\cos\theta \qquad C = L\sin\theta$

$x: \quad 0 = F_f - F_{N2}$

$F_{N2} = F_f$

$y: \quad 0 = F_{N1} - F_{g1} - F_{g2}$

#1 $F_{N1} = (m_1 + m_2)g$

#2 $F_{N2} = \mu F_{N1} = \mu(m_1 + m_2)g$

$\tau: \quad 0 = (A)(F_{g1}) + (B)(F_{g2}) - (C)(F_{N2})$

$0 = (\frac{L}{2}\cos\theta)(m_1 g) + (D\cos\theta)(m_2 g) - (L\sin\theta)(\mu)(m_1 + m_2)g$

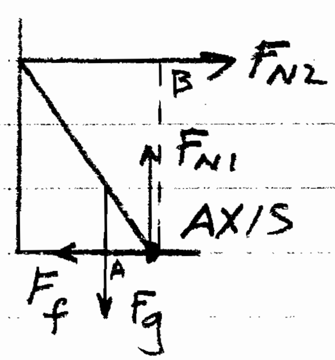
#3

$$\therefore D = L\left[\frac{\mu(m_1 + m_2)}{m_2}\tan\theta - \frac{m_1}{2m_2}\right]$$

EXAMPLE: A 100 lb LADDER, WHICH IS 20′ LONG, IS INCLINED AT 60°. ITS CENTER OF GRAVITY IS 8′ FROM ITS BASE. FIND μ SO THAT THE LADDER IS SAFE.

LEMMA: $A = 8\cos 60° = 4$

$B = 20\sin 60° = 17.3$



$y: \quad 0 = F_{N1} - F_g \qquad F_{N1} = 100$

$\tau: \quad 0 = (B)(F_{N2}) - (A)(F_g)$

$0 = (17.3)F_{N2} - (4)(100) \qquad F_{N2} = 23$

$x: \quad 0 = F_{N2} - F_f$

$0 = 23 - F_f \qquad F_f = 23$

LEMMA: $F_f = \mu F_{N1}$

$23 = \mu(100) \qquad \mu = .23$