GRAVITATION

I. NEWTON'S LAW OF UNIVERSAL GRAVITATION

A. FORMULA: $F_g = \dfrac{G m_1 m_2}{r^2}$

m_1 AND m_2 = ANY TWO MASSES [KG]
r = DISTANCE BETWEEN THEIR CENTERS [m]
$G = 6.67 \times 10^{-11}$ [Nm^2/KG^2]
F_g = FORCE OF ATTRACTION [N]

B. GRAVITY IS AN INVERSE SQUARE LAW BECAUSE OF r^2 IN THE DENOMINATOR.

CHANGE IN:

DISTANCE		FORCE	
2X	CLOSER	4X	STRONGER
3X		9X	
4X	FURTHER	16X	WEAKER
5X		25X	

II. GRAVITY ON OTHER PLANETS

A. $g_{PLANET} = \dfrac{GM}{R^2} = [m/s^2]$ ALWAYS (+).

g = PLANET'S INFLUENCE IN CREATING OUR WEIGHT.
M = MASS [kg] } OF THE PLANET.
R = RADIUS [m] }
$G = 6.67 \times 10^{-11}$ [Nm^2/kg^2]

B. JUSTIFICATION

WEIGHT FORMULA

$F_g = mg$

m = OUR MASS.

NEWTON'S LAW OF GRAVITY.

$F_g = GmM/R^2$

$\therefore \; mg = GmM/R^2$

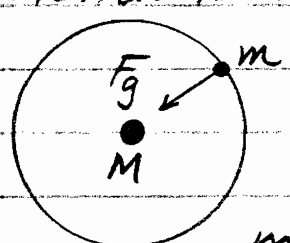
$\therefore g = GM/R^2$.

C. EXAMPLE: EARTH. GIVEN: $R = 6378000$ m
$M = 5.98 \times 10^{24}$ KG. FIND g FOR EARTH.

$$g = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6378000)^2} = 9.8 \text{ m/s}^2$$

III. ORBITAL MOTION

A. THE MOON: m = MOON'S MASS. IT IS THE OBJECT WHICH IS ACCELERATING BECAUSE ITS DIRECTION OF MOTION IS CHANGING. M = MASS OF EARTH



r = DISTANCE FROM EARTH TO MOON.
FIND THE TIME FOR THE MOON TO ORBIT THE EARTH.

$$ma = F_g$$

$$m(v^2/r) = GmM/r^2$$

$$v^2 = GM/r$$

LEMMA

$$v = \frac{2\pi r}{T}$$

#1

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

#2 $$T^2 = \frac{4\pi^2 r^3}{GM}$$

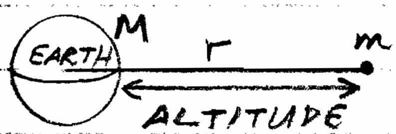
$$T = \left[\frac{4\pi^2(3.8\times10^8)^3}{(6.67\times10^{-11})(5.98\times10^{24})}\right]^{1/2}$$

$$T = 2.33\times10^6 \text{ SEC} = 27 \text{ DAYS}$$

B, VARIATIONS IN THE SATELLITE FORMULA:

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2} \qquad r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \qquad M = \frac{4\pi^2 r^3}{GT^2}$$

C, GEOSTATIONARY SATELLITE: FIND THE ALTITUDE OF A SATELLITE WHICH ORBITS THE EARTH ONCE EVERY TWENTY-FOUR HOURS. SATELLITES CLOSE TO THE SURFACE OF THE EARTH ORBIT TOO QUICKLY TO MATCH THE EARTH'S SLOW SPIN RATE.



$T = 24$ HRS $= 86{,}400$ SEC.

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11})(5.98\times10^{24})(86400)^2}{4\pi^2}\right]^{1/3}$$

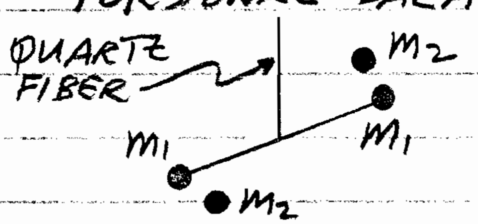
$r = 4.225\times10^{7}$ m $= 42250$ KM

RADIUS OF EARTH $= \underline{6378\text{ KM}}$ −

H = ALTITUDE $= 35{,}872$ KM $= 22{,}400$ miles

IV. LORD HENRY CAVENDISH

A. 1798: MEASURED $G = 6.67\times10^{-11}$ USING A TORSIONAL BALANCE. DEVICE: THE BAR TWISTS IN A HORIZONAL PLANE UNTIL THE RETARDING TWIST IN THE FIBER MATCHES THE TWIST DUE TO F_g BETWEEN m_1 AND m_2



ANALYSIS: $F_g = G m_1 m_2 / r^2$

m_1 AND m_2 ARE MEASURED WITH A BALANCE.

r, THE DISTANCE FROM m_1 TO m_2, IS MEASURED USING A METER STICK.

F_g IS MEASURED BY KNOWING THE STIFFNESS OF THE FIBER AND THE ANGLE THAT IT TWISTS.

∴ CAVENDISH COULD EXPERIMENTALLY CALCULATE THE VALUE OF G.

B. COROLLARY: CAVENDISH IS THE "FIRST PERSON TO FIND THE MASS OF THE EARTH." $g = GM/R^2$.

$10 = (6.67 \times 10^{-11})\, M \,/\, (6378000)^2$

$\therefore M = 5.98 \times 10^{24}$ kg

V. INTUITIONS FOR GRAVITY

A. $g = GM/R^2$ $\qquad F_g = mg$

OUR MASS DOES NOT CHANGE ON OTHER PLANETS.

RECIPE: 1) ON EARTH, $g = 10\ m/s^2$.

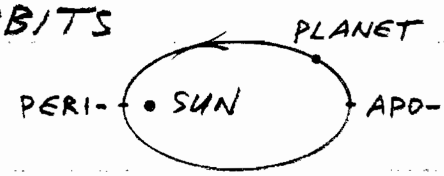
2) WRITE A CHAIN STARTING WITH $10\ m/s^2$ WITH LINKS TO ACCOUNT FOR THE MASS AND RADIUS CHANGES. EXAMPLE: BLAKE'S MASS IS 80 KG. HE VISITS A NEW PLANET NEON, WHICH HAS FOUR TIMES THE MASS AND FIVE TIMES THE RADIUS OF EARTH. FIND g ON NEON AND BLAKE'S WEIGHT.

$g = 10\left(\frac{4}{1}\right)\left(\frac{1}{5}\right)^2 = 1.6\ m/s^2 \qquad F_g = mg = (80)(1.6) = 128\,N$

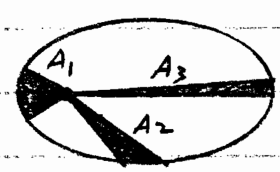
VI. KEPLER'S LAWS OF PLANETARY MOTION

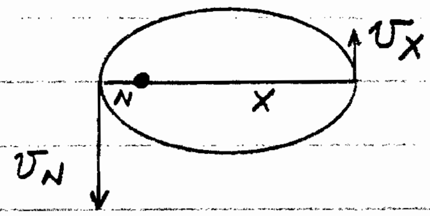
A. 1ST: LAW OF ELLIPTICAL ORBITS

THE PLANETS ORBIT COUNTER-CLOCKWISE AROUND THE SUN, WHICH IS LOCATED AT ONE OF THE FOCII.



B. 2ND: LAW OF EQUAL AREAS: IN A GIVEN TIME, A PLANET WILL SWEEP OUT EQUAL AREAS. COROLLARY: THE PLANET TRAVELS QUICKEST NEAR THE SUN.





$N v_N = X v_X$

$A_1 = A_2 = A_3$

C. 3RD: LAW OF HARMONY OR LAW OF PERIODS

$$\left(\frac{T^2}{R^3}\right)_A = \left(\frac{T^2}{R^3}\right)_B$$

T = PERIOD

R = AVERAGE RADIUS OF THE ORBIT

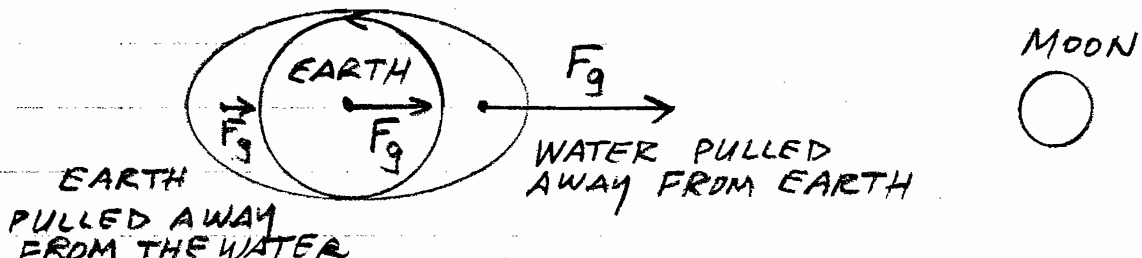
A AND B ARE TWO SATELLITES WHICH ORBIT THE SAME BODY. FOR PLANETS, LET BODY B ALWAYS BE THE EARTH. EXAMPLE: URANUS ORBITS THE SUN EVERY 84 YEARS. FIND ITS AVERAGE DISTANCE TO THE SUN. A.U. = ASTRONOMICAL UNIT.

$$\frac{R^3}{(84\text{ YEARS})^2} = \frac{(1\text{ A.U})^3}{(1\text{ YEAR})^2} \qquad R = 19\text{ A.U.}$$

VII. TIDES

A. LUNAR EFFECTS

LOOKING DOWN ON NORTH POLE OF EARTH.

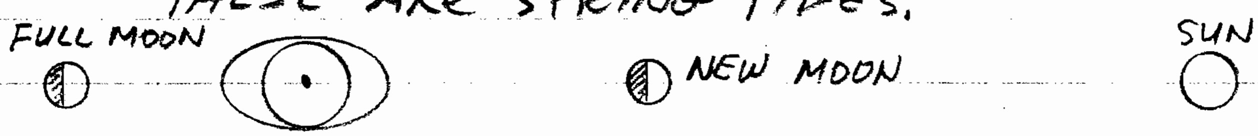


AS THE EARTH SPINS ON ITS AXIS, EACH TIME WE PASS A BULGE IN THE WATER, WE EXPERIENCE HIGH TIDE.

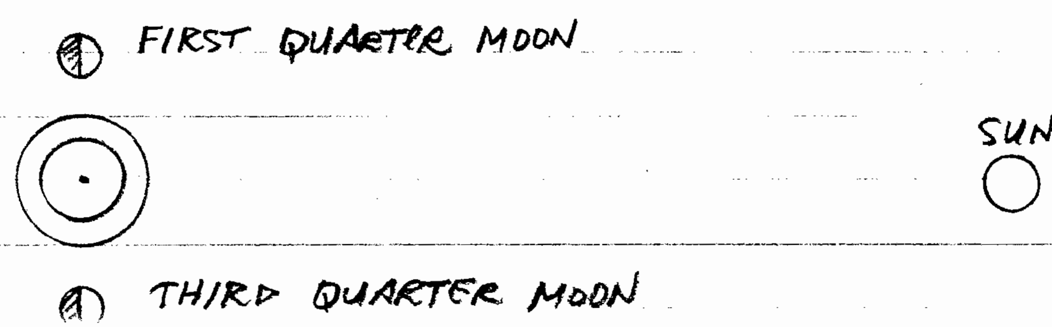
$$\frac{\text{MOON EFFECT}}{\text{SUN EFFECT}} = 9/4$$

B. SOLAR EFFECTS

WHEN THE SUN AND THE MOON ARE ALIGNED, THE TIDES ARE EXTREMELY HIGH AND EXTREMELY LOW. THESE ARE SPRING TIDES.



WHEN THE SUN AND THE MOON ARE AT RIGHT ANGLES, THE TIDES ARE NOT EXTREME. THESE ARE NEAP TIDES.

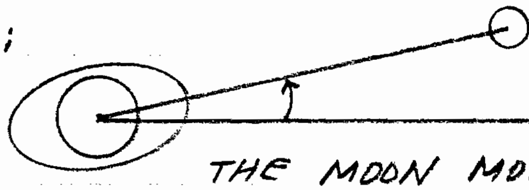


C. DUE TO THE ORBITING OF THE MOON COUNTER-CLOCKWISE AROUND THE EARTH, THE HIGH TIDE COMES ABOUT ONE HOUR LATER EACH DAY.

TODAY:

MOON

TOMORROW:



THE MOON ORBITS THE EARTH IN 27.3 DAYS

THE MOON MOVED $1/27^{th}$ OF ITS ORBIT. ∴ EARTH MUST DO $1/27^{th}$ OF A SPIN TO CATCH THE HIGH TIDE BULGE OF WATER.

$$\frac{1}{27.3}\,(1\ \text{DAY})\left(\frac{24\ \text{HR}}{\text{DAY}}\right)\left(\frac{60\ \text{MIN}}{\text{HR}}\right) = 53\ \text{MINUTES.}$$