A. KINEMATICS

1. CONVERT THE FOLLOWING INTO S.I. UNITS.

a) 270° = _____ RADIANS

b) 15 REV. = _____ RADIANS

c) 3000 RPM = _____ SEC^{-1}

d) 1 REV/DAY = _____ SEC^{-1}

e) $240^\circ/HR$ = _____ S^{-1}

f) 1600 RPM/S = _____ S^{-2}

(4.7, 94.2, 314, 7.27×10^{-5}, 1.1636×10^{-3}, 167.6)

2. THE PROP ON SHIRLEY'S CESSNA® ACCELERATES FROM 800 RPM TO 3800 RPM IN FIVE MINUTES. FIND THE NUMBER OF REVOLUTIONS MADE IN THAT TIME. (11500)

3. THE CD PLAYER IS TURNED ON. MOZART ACCELERATES FROM REST TO 5.2 SEC^{-1} IN .4 SECONDS. FIND:

A) HIS ANGULAR ACCELERATION. ($13\ SEC^{-2}$)

B) HIS ANGULAR DISPLACEMENT. (1.04 RADIANS)

4. AS MR. SKINNER'S YO-YO FALLS, IT ACCELERATES FROM REST TO 60 SEC^{-1} AT 24 SEC^{-2}. FIND:

A) THE ANGULAR DISPLACEMENT (75 RADIANS)

B) THE TIME THAT IT FELL. (2.5 SEC)

5. THE TIRES ON KATHY'S WHITE BMW ARE ROTATING AT ANGULAR SPEED 100 SEC^{-1}. SCHOOL ZONE! KATHY SLOWS DOWN A BIT AT $-6\ SEC^{-2}$ OVER AN ANGULAR DISPLACEMENT OF 425 RADIANS. FIND:

A) THE TIME INTERVAL OVER WHICH SHE BRAKED. (5 SEC)

B) HER FINAL ANGULAR VELOCITY. ($70\ SEC^{-1}$)

6. IN PROBLEM 4, THE SPOOL OF THE YO-YO HAS A RADIUS OF .008 m. FIND: A) ITS FINAL LINEAR SPEED.

B) THE TOTAL DISTANCE FALLEN (.48 m/s, .6 m)

7. IN PROBLEM 5, THE TIRES HAVE A RADIUS OF .36 m. FIND: A) HER INITIAL LINEAR VELOCITY

B) LINEAR ACCELERATION C) FINAL VELOCITY

D) TOTAL DISTANCE TRAVELED (36 m/s, $-2.16\ m/s^2$, 25.2, 153)

8. ATLANTA 1996! JOHN IS DOING THE HAMMER THROW, WHICH CONSISTS OF A 7.8 KG MASS TIED TO THE END OF A CHAIN .6 m LONG. JOHN GRABS THE FREE END OF THE CHAIN AND WHIRLS AROUND, ACCELERATING THE MASS FROM 0 TO 30 m/s IN 1.2 SECONDS IN A CIRCLE. FIND:
A) THE TANGENTIAL ACCELERATION. ($25\ m/s^2$)
B) THE ANGULAR ACCELERATION. ($41.66\ sec^{-2}$)
WE FREEZE-FRAME HIS EVENT .24 SECONDS AFTER HE FIRST STARTED TO WHIRL AROUND.
FIND: C) THE LINEAR VELOCITY OF THE MASS (6 m/s)
D) THE ANGULAR VELOCITY ($10\ sec^{-1}$)
E) THE FREQUENCY ($1.59\ H_3$)
F) RADIAL ACCELERATION ($60\ m/s^2$)
G) TOTAL ACCELERATION ($65\ m/s^2$ AT 67.38°)
H) TOTAL FORCE ON THE MASS (507N AT 67.38°)

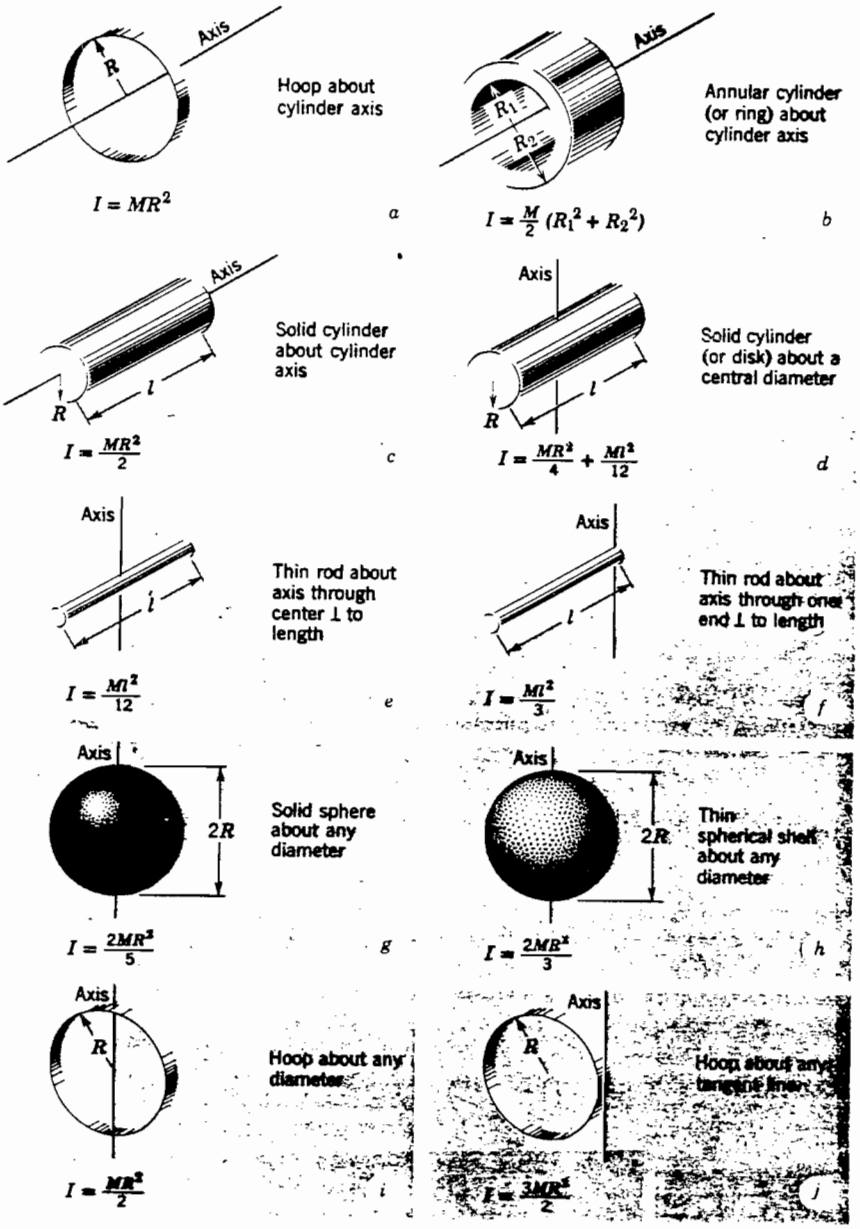
9. FIND THE ANGULAR VELOCITY, WHICH IS CONSTANT, OF THE SECOND HAND OF A SWATCH.® ($.1047\ sec^{-1}$)

10. FIND THE AVERAGE ANGULAR VELOCITY OF THE EARTH AS IT ORBITS THE SUN. ($2 \times 10^{-7}\ sec^{-1}$)

11. THREE HUNDRED FORTY MILLION YEARS AGO THERE WERE 400 DAYS PER YEAR. SPINNING QUICKER THAN PRESENTLY, THE EARTH COMPLETED ONE ROTATION ON ITS AXIS IN ONLY 22 HOURS. FIND: A) THE ANCIENT AND THE PRESENT ANGULAR SPEED OF THE EARTH'S ROTATION ON ITS AXIS. (7.93×10^{-5}, $7.27 \times 10^{-5}\ sec^{-1}$)
B) THE ANGULAR DECELERATION OF THE EARTH'S SPIN. ($-6.16 \times 10^{-22}\ sec^{-2}$)

MOMENT OF INERTIA

$$I = \int (r^2)\, dm \quad [KG \cdot m^2]$$

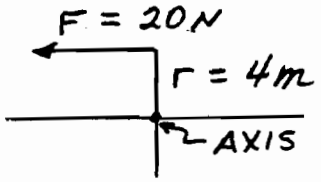


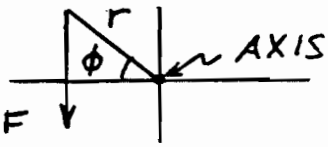
$I_{\text{POINT MASS}} = mr^2$

PARALLEL AXIS THEOREM : $I = I_{C.M.} + mh^2$

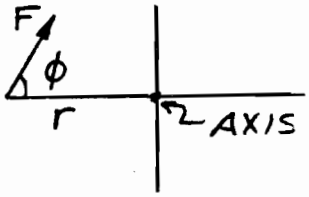
B. DYNAMICS

1. FIND THE MAGNITUDE AND THE DIRECTION OF THE TORQUE FOR EACH OF THE FOLLOWING:

A) 

B) 

$\phi = 58.11^\circ$
$r = 106\ m$
$F = 3\ N$

C) 

$\phi = 59.49^\circ$
$F = 65\ N$
$r = 4\ m$

ANSWERS:
(80 Nm, OUT OF THE PAGE ⊙; 168, ⊙; 224 ⊗)

2. THE EARTH HAS MASS = 5.98×10^{24} KG AND RADIUS = 6.4×10^{6} m. FIND THE MOMENT OF INERTIA FOR THE EARTH AS IT SPINS ON ITS AXIS. ($9.8 \times 10^{37}\ kg m^2$)

3. FIND THE MOMENT OF INERTIA FOR THE ORBITAL MOTION OF THE EARTH ABOUT THE SUN, WHICH IS 1.496×10^{11} m AWAY FROM US. ($1.34 \times 10^{47}\ kg m^2$)

4. A BICYCLE RIM, WHOSE MASS IS .8KG AND WHOSE RADIUS IS .3m, IS HUNG ALONG IS EDGE BY A NAIL IN THE WALL. FIND ITS MOMENT OF INERTIA. (.144)

5. A HOLLOW SAPPHIRE GLOBE, WHOSE $m = 6$ KG AND $r = 1.5$ m, IS HUNG BY VALERIE FROM THE CEILING WITH CARE. THE GLOBE IS TANGENT TO THE CEILING. FIND ITS MOMENT OF INERTIA. ($22.5\ kg m^2$)

6. SERGEI BUBKA, $m = 70$ KG, ROTATES ON THE END OF A 5m LONG, 2.4KG POLE, AS HE VAULTS FOR THE GOLD MEDAL. FIND THE MOMENT OF INERTIA FOR: A) HIS BODY (1750) B) THE POLE (20) C) THE ENTIRE ATHLETIC SYSTEM (1770)

7. A PLAYGROUND CAROUSEL IS A SOLID DISK OF MASS 48 KG AND RADIUS 4 m. TO ACCELERATE THE MERRY-GO-ROUND FROM REST, RIGEL APPLIES A FORCE OF 360 N TO ITS EDGE. FIND: A) ANGULAR ACCELERATION B) THE TANGENTIAL ACCELERATION OF A POINT ON THE RIM. ($3.75\ sec^{-2}$, $15\ m/s^2$)
AT $t = .8$ SECONDS AFTER THE CAROUSEL WAS AT REST, FIND THE: C) ANGULAR VELOCITY D) LINEAR VELOCITY OF A POINT ON THE RIM E) RADIAL ACCELERATION F) TOTAL ACCELERATION G) ANGULAR DISPLACEMENT H) DISTANCE TRAVELED BY THE RIM.
($3\ sec^{-1}$, $12\ m/s$, $36\ m/s^2$, $39\ m/s^2$ AT $67.38°$, 1.2 RAD, $4.8\ m$)

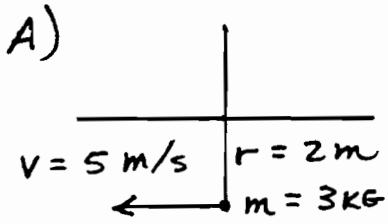
8. JENNY'S POTTER'S WHEEL, WITH RADIUS .18 m. AND MOMENT OF INERTIA $3\ kg\,m^2$, IS INITIAL ROTATING AT ANGULAR VELOCITY $550\ sec^{-1}$! TO SLOW IT DOWN, JENNY PUTS THE PALM OF HER HAND ON THE RIM AND EXERTS A BRAKING FORCE OF 200N FOR 5 SECONDS. FIND THE FINAL ANGULAR SPEED. ($490\ sec^{-1}$)

9. KAI HAS A GOLD GLOBE WHOSE MASS IS 1260 kg AND WHOSE RADIUS IS .25 m. SHE MOUNTS IT ON A SPINDLE SO THAT IT CAN ROTATE FREELY. KAI WRAPS A STRING AROUND THE EQUATOR OF THE GLOBE. SHE NOW APPLIES A 42 N FORCE TO THE ROPE FOR SIX SECONDS. AS A RESULT, THE GLOBE ACCELERATES FROM REST TO AN ANGULAR SPEED OF $2\ sec^{-1}$.
IS THE GLOBE A SUBSTANTIAL SOLID ONE OR IS IT HOLLOW?

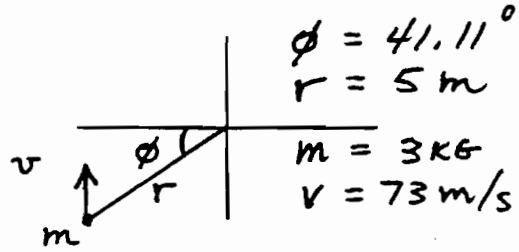
C. Angular Momentum

1. Find the magnitude and the direction of the angular momentum for each of the following:

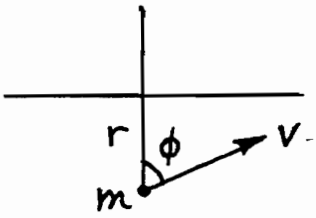
A)



B)



C)



m = 3kg
V = 26 m/s
r = 5 m
φ = 67.38°

ANSWERS:
(30 kg m²/s, ⊗ into the page, 825 ⊗, 360 ⊙)

2. "Please shut the door," Mr. Harvie asks. Spencer smacks the door .5 m from the frictionless hinges. His fist, in contact with the door for only .1 sec, exerts a perpendicular force of 192 N. The door has a moment of inertia of 12 kg m². Find:
A) the angular speed with which the door leaves his hand. (.8 sec⁻¹) B) the time for the door to rotate at this constant angular speed through an angular displacement of π/2 radians. (1.96 sec)

3. A grindstone, which is a solid disk of m = 3kg and radius .08 m, is spinning freely at angular speed 220 sec⁻¹! Putting his nose to the grindstone, Mr. H. exerts a force of 24 N to the edge of the stone for .7 seconds. Find the final ω. (80 sec⁻¹)

4. NAGANO, JAPAN 1998! NAO, THE FIGURE SKATER, SPINS AT 8 sec^{-1} WITH HER ARMS EXTENDED, SO THAT HER MOMENT OF INERTIA IS 21 kgm^2. SHE PULLS IN HER ARMS REDUCING I TO 7 kgm^2. GOLD MEDAL! FIND:
A) HER FINAL ANGULAR SPEED. (24 sec^{-1})
B) THE ENERGY SUPPLIED BY HER MUSCLES. (1344 JOULES)

D. CONSERVATION OF ENERGY

1. A SOLID DISK, A SOLID BALL AND A HOLLOW SPHERE START AT REST AT THE SAME ELEVATION. THEY RACE DOWN AN INCLINE, ROLLING WITHOUT SLIPPING. NAME THE ORDER OF FINISH.
2. THEY RACE AGAIN BUT THIS TIME THERE IS NO FRICTION SO THAT THEY SLIDE DOWN. WHO WINS?
3. CALCULATE THE FINAL SPEED OF A HOOP ROLLING DOWN AN INCLINE WHOSE ELEVATION IS 4.9m. (7 m/s)
4. A GENERIC SOLID IS ROLLING ALONG A FLAT HORIZONTAL SURFACE AT SPEED v_0. A HILL! FIND A FORMULA FOR THE MAXIMUM ELEVATION ATTAINED BY THIS SISYPHEAN SOLID. ($\frac{v_0^2(1+k)}{2g}$)
5. A SOLID DISK, A SOLID BALL AND A HOLLOW SPHERE ARE ROLLING ALONG THE FLAT ROAD AT SPEED v_0. UP THE HILL! WHO GOES HIGHEST? MEDIUM? LOWEST?

6. THEY ARE RACING AGAIN ALONG THE FLAT SECTION. ALL HAVE SPEED v_0, BUT THIS TIME THEY ARE SLIDING WITHOUT ROLLING. UP THE HILL! WHO GOES HIGHEST?

7. A HOOP IS ROLLING WITHOUT SLIPPING AT 15 m/s ALONG THE HORIZONTAL ROAD. UP THE INCLINE! FIND ITS MAXIMUM ELEVATION. (22.5m)

8. JENNIFER HAS A ROUND CHOCOLATE VALENTINE CANDY. STARTING IT FROM REST AT AN ELEVATION OF 63m, SHE ROLLS IT WITHOUT SLIPPING DOWN AN INCLINED PLANE. ITS FINAL VELOCITY IS 30m/s. IS THIS VALENTINE A DELICIOUS SOLID ONE OR IS IT A DISAPPOINTING HOLLOW ONE?

REVIEW PROBLEMS

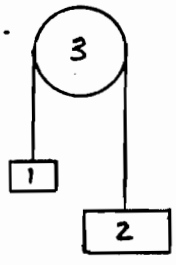
1. SUMMER TIME! MR. HARVIE, A SOLID CYLINDER OF $m = 60$KG AND $R = .2$m, ROLLS DOWN A GRASSY KNOLL OF ELEVATION 30m. THE HILL IS INCLINED AT 36.87°. USE DYNAMICS TO FIND:

A) THE ACCELERATION DOWN THE INCLINE. ($4\ m/s^2$)

B) THE TIME TO REACH THE BOTTOM. (5 SECONDS)

C) THE FORCE OF FRICTION BETWEEN ME AND THE LAWN. (120N)

D) THE MINIMUM COEFFICIENT OF FRICTION NECESSARY TO KEEP ME ROLLING WITHOUT SLIPPING. (.25)

2. 

THE SHEAVE OF THE PULLEY IS A SOLID DISK WITH $m_3 = 48$ KG.

$m_1 = 16$ KG $\quad m_2 = 60$ KG

FIND THE ACCELERATION OF THE MASSES AND THE TENSION IN THE ROPE ON EACH SIDE OF THE PULLEY. ($4.4\ m/s^2$, 230.4N, 336N)