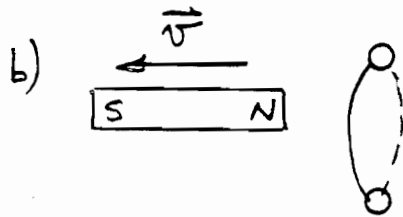
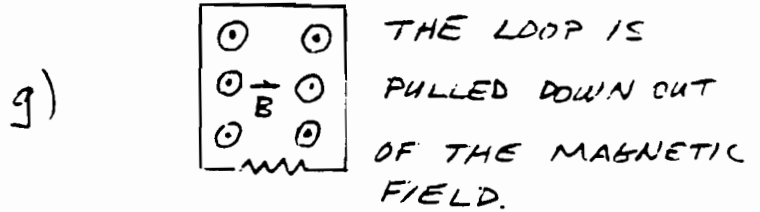
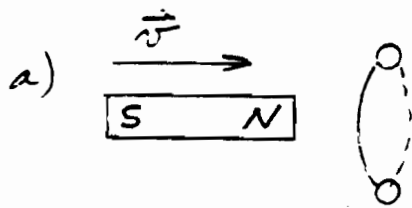
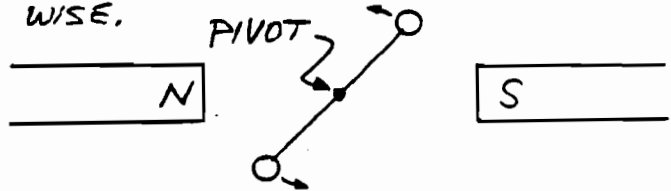


# FARADAY AND LENZ GET TOGETHER TO MAKE JUICE 69

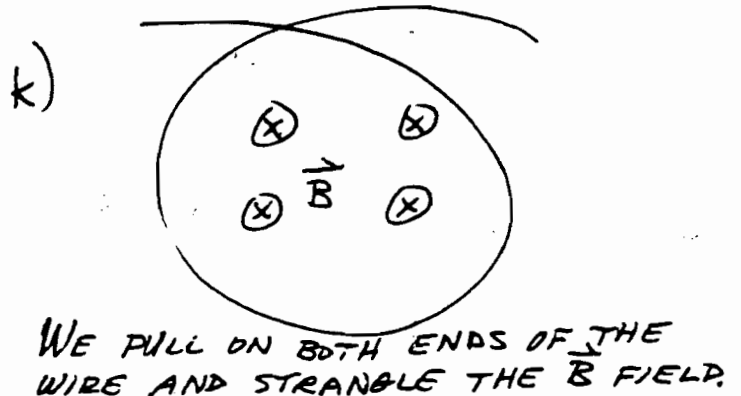
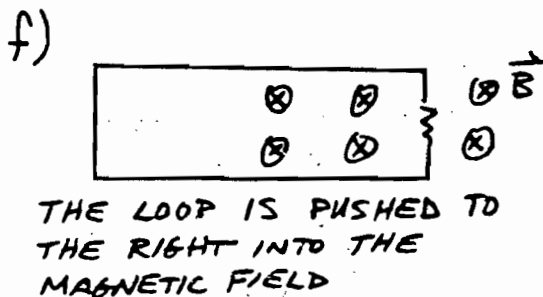
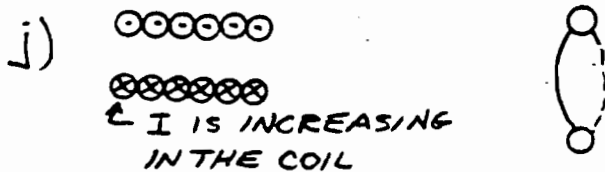
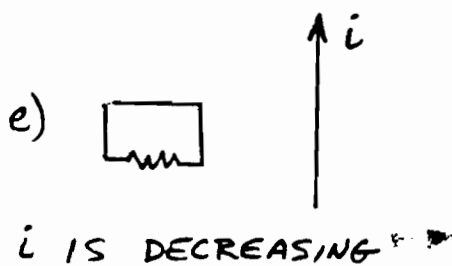
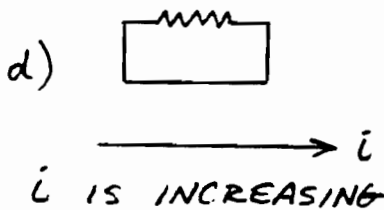
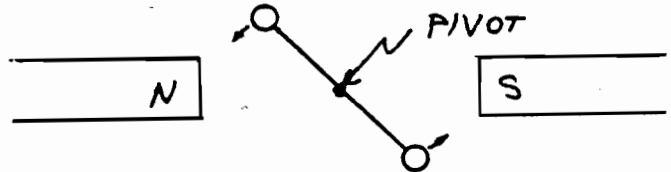
ON THE LOOP, INDICATE THE DIRECTION OF THE INDUCED CURRENT.



h) WE ARE LOOKING DOWN ON A LOOP OF WIRE WHICH CAN SPIN IN A  $\vec{B}$  FIELD. INITIALLY, THE LOOP WAS PARALLEL TO THE FIELD SO THAT IT POSSESSED NO FLUX. WE ARE SPINNING THE LOOP COUNTERCLOCKWISE.



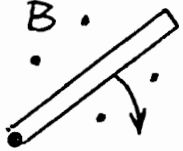
i) INITIALLY, THE LOOP WAS PERPENDICULAR TO THE  $\vec{B}$  FIELD. WE ARE NOW SPINNING THE LOOP C.C.W.



## FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

1. A CONDUCTING BAR .5 m LONG IS PUSHED AT SPEED 8 m/s THROUGH A CONSTANT MAGNETIC FIELD OF STRENGTH 1.5 TESLA. FIND: a) THE VOLTAGE INDUCED BETWEEN THE TWO ENDS, b) THE ELECTRIC FIELD CREATED INSIDE THE BAR. (6V, 12 V/m)

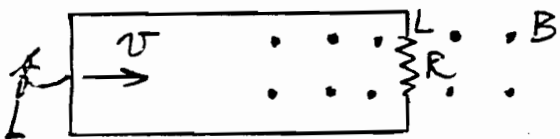
2. A CONDUCTING BAR .5 m LONG IS ROTATED ABOUT ONE END THROUGH A MAGNETIC FIELD OF 3 TESLA. THE BAR PERFORMS FIVE ROTATIONS IN 1.309 SEC. FIND: a) LABEL THE (+) AND THE (-) ENDS OF THE BAR.



b) THE VOLTAGE INDUCED BETWEEN THE ENDS OF THE BAR. (9V)

c) FIND THE ELECTRIC FIELD INDUCED INSIDE THE BAR. ( $18 \frac{V}{m}$ )

3. A LOOP OF WIRE IS PUSHED INTO A REGION OF CONSTANT MAGNETIC FIELD. GIVEN:  $B = 4T$   $L = 5m$   $v = 6m/s$   $R = 15 \Omega$



FIND: a) THE DIRECTION OF THE INDUCED CURRENT.

b) THE ELECTROMOTIVE FORCE [VOLTS] DRIVING THE CURRENT.

c) THE ELECTRIC FIELD IN EACH OF THE SEGMENTS OF THE LOOP.

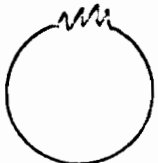
d) THE CURRENT THROUGH THE RESISTOR.

e) THE ELECTRICAL POWER DELIVERED TO THE RESISTOR.

f) THE FORCE SUPPLIED BY THE PHYSICIST.

(CLOCKWISE; 120V; 0, 0,  $24 \frac{V}{m}$ , 0; 8A, 960W, 160N)

4. A LOOP OF WIRE HAS AREA .125  $m^2$  AND RESISTANCE OF  $7 \Omega$ .  $\vec{B}$  IS PERPENDICULAR TO THE PAGE.



VECTORS INTO THE PAGE ARE MINUS.

$$B = 84 t^2 - 336 t$$

a) FIND A FORMULA FOR  $dB/dt$ .

b) FIND THE DIRECTIONS OF THE INDUCED CURRENT.

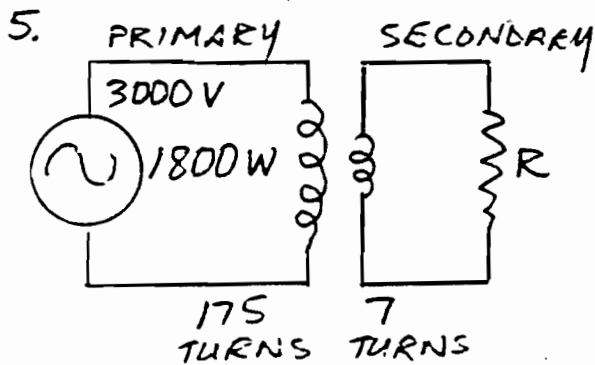
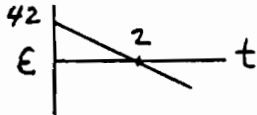
c) FIND A FORMULA FOR THE E.M.F.,  $\epsilon$ , IN [VOLTS].

d) At  $t = 5$ , FIND THE VALUES OF THE E.M.F., THE CURRENT AND THE POWER DISSIPATED BY THE RESISTOR.

e) SKETCH A GRAPH OF  $B$  VERSUS  $t$  AND  $E$  VS.  $t$ .

$$\left( \frac{dB}{dt} = 168t - 336; \text{ CCW UNTIL } t=2, \text{ THEN C.W.}; \right.$$

$$E = -21t + 42, -63 \text{ V}, 9 \text{ A c.w.}, 567 \text{ W}, \left. \begin{array}{l} B \\ 2 \\ 4 \end{array} \right| t$$



- FIND:
- CURRENT FLOWING IN THE PRIMARY CIRCUIT
  - VOLTAGE OF SECONDARY CIRCUIT
  - CURRENT FLOWING IN SECONDARY
  - RESISTANCE  $R$  IN OHMS.
  - POWER DELIVERED TO THE RESISTOR.

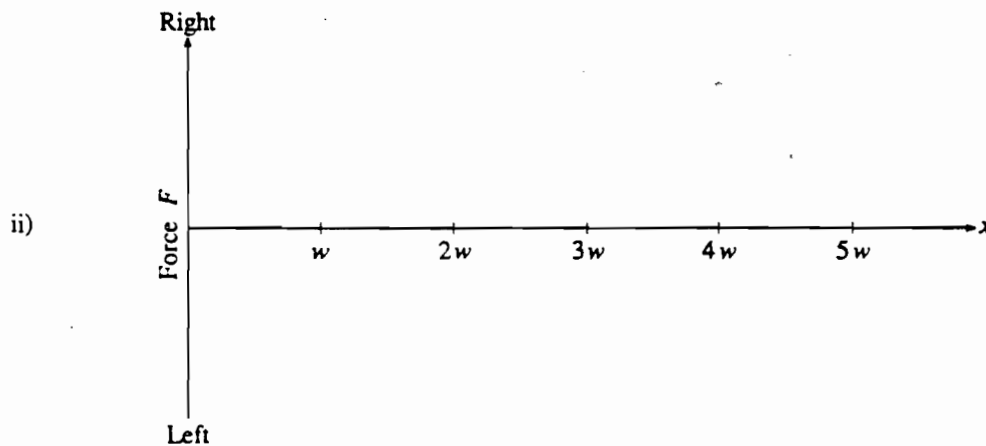
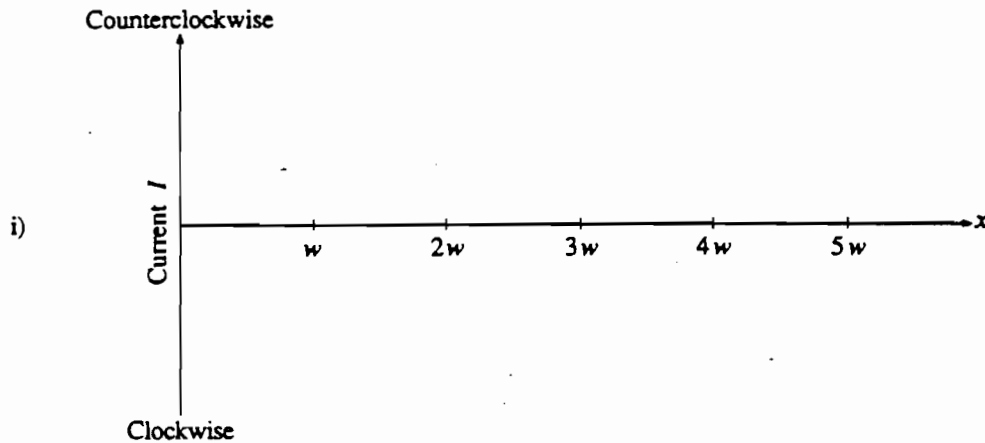
(.6, 120, 15, 8, 1800)



(b) On the axes below, plot the following as functions of position  $x$  of the right edge of the loop shown above.

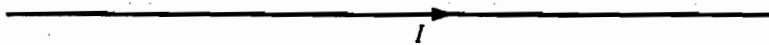
- i) The induced current  $I$  in the loop
- ii) The applied force  $F$  required to keep the loop moving at constant speed

Let counterclockwise current be positive, clockwise current be negative, forces to the right be positive, and forces to the left be negative. The graphs should begin with the loop in the position shown ( $x = 0$ ) and continue until the right edge of the loop is a distance  $2w$  to the right of the region containing the field ( $x = 5w$ ).



GO ON TO THE NEXT PAGE 

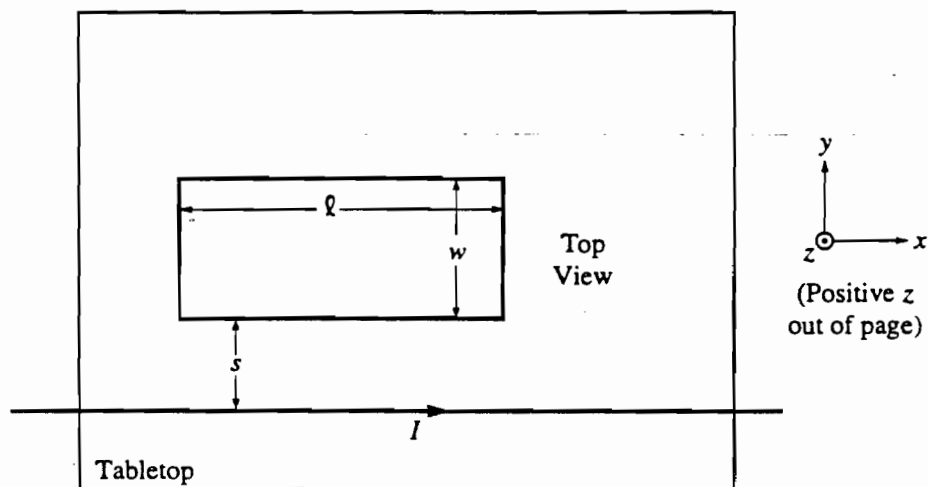




E & M 3. A long, straight wire lies on a table and carries a constant current  $I$ , as shown above.

- (a) Using Ampere's law, derive an expression for the magnitude  $B$  of the magnetic field at a perpendicular distance  $r$  from the wire.

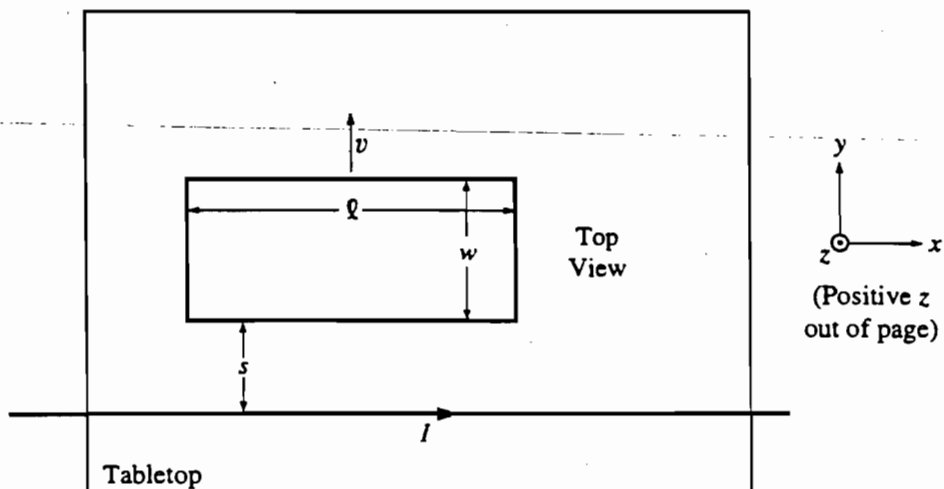
A rectangular loop of wire of length  $\ell$ , width  $w$ , and resistance  $R$  is placed on the table a distance  $s$  from the wire, as shown below.



- (b) What is the direction of the magnetic field passing through the rectangular loop relative to the coordinate axes shown above on the right?

GO ON TO THE NEXT PAGE

- (c) Show that the total magnetic flux  $\phi_m$  through the rectangular loop is  $\frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{s+w}{s}\right)$ .

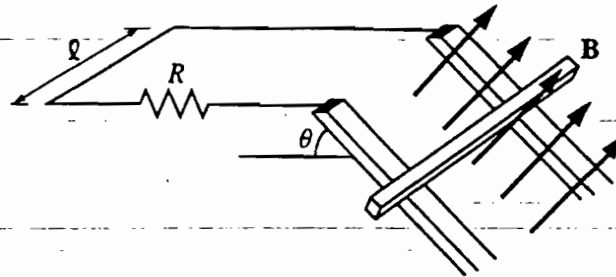


The rectangular loop is now moved along the tabletop directly away from the wire at a constant speed  $v = \left| \frac{ds}{dt} \right|$ , as shown above.

- (d) What is the direction of the current induced in the loop? Briefly explain your reasoning.
- (e) What is the direction of the net magnetic force exerted by the wire on the moving loop relative to the coordinate axes shown above on the right? Briefly explain your reasoning.
- (f) Determine the current induced in the loop. Express your answer in terms of the given quantities and fundamental constants.

**STOP**

END OF SECTION II, ELECTRICITY AND MAGNETISM



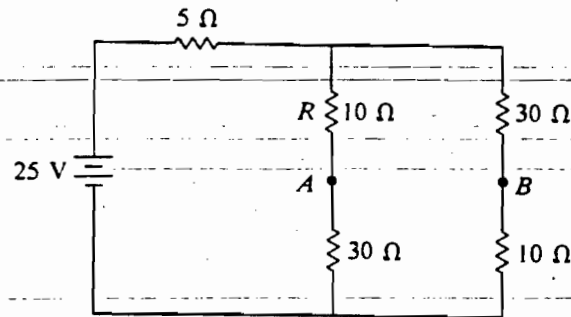
E & M. 3. A conducting bar of mass  $m$  is placed on two long conducting rails a distance  $l$  apart. The rails are inclined at an angle  $\theta$  with respect to the horizontal, as shown above, and the bar is able to slide on the rails with negligible friction. The bar and rails are in a uniform and constant magnetic field of magnitude  $B$  oriented perpendicular to the incline. A resistor of resistance  $R$  connects the upper ends of the rails and completes the circuit as shown. The bar is released from rest at the top of the incline. Express your answers to parts (a) through (d) in terms of  $m$ ,  $l$ ,  $\theta$ ,  $B$ ,  $R$ , and  $g$ .

- Determine the current in the circuit when the bar has reached a constant final speed.
- Determine the constant final speed of the bar.
- Determine the rate at which energy is being dissipated in the circuit when the bar has reached its constant final speed.
- Express the speed of the bar as a function of time  $t$  from the time it is released at  $t = 0$ .
- Suppose that the experiment is performed again, this time with a second identical resistor connecting the rails at the bottom of the incline. Will this affect the final speed attained by the bar, and if so, how? Justify your answer.

**STOP**

END OF SECTION II, ELECTRICITY AND MAGNETISM





E&M 2. Five resistors are connected as shown above to a 25-volt source of emf with zero internal resistance.

(a) Determine the current in the resistor labeled  $R$ .

A 10-microfarad capacitor is connected between points  $A$  and  $B$ . The currents in the circuit and the charge on the capacitor soon reach constant values. Determine the constant value for each of the following.

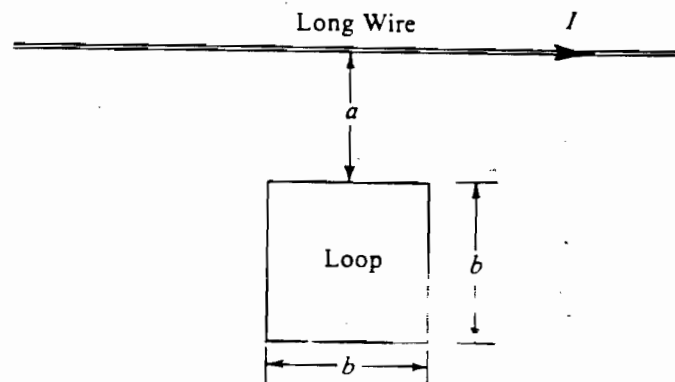
(b) The current in the resistor  $R$

(c) The charge on the capacitor

The capacitor is now replaced by a 2.0-henry inductor with zero resistance. The currents in the circuit again reach constant values. Determine the constant value for each of the following.

(d) The current in the resistor  $R$

(e) The current in the inductor



E&M 3. A long wire carries a current in the direction shown above. The current  $I$  varies linearly with time  $t$  as follows.

$$I = ct, \text{ where } c \text{ is a positive constant}$$

The long wire is in the same plane as a square loop of wire of side  $b$ , as shown in the diagram. The side of the loop nearest the long wire is parallel to it and a distance  $a$  from it. The loop has a resistance  $R$  and is fixed in space.

(a) Determine the magnetic field  $B$  at a distance  $r$  from the long wire as a function of time.

(b) Indicate on the diagram the direction of the induced current in the loop.

(c) Determine the induced current in the loop.

(d) State whether the magnetic force on the loop is toward or away from the wire.

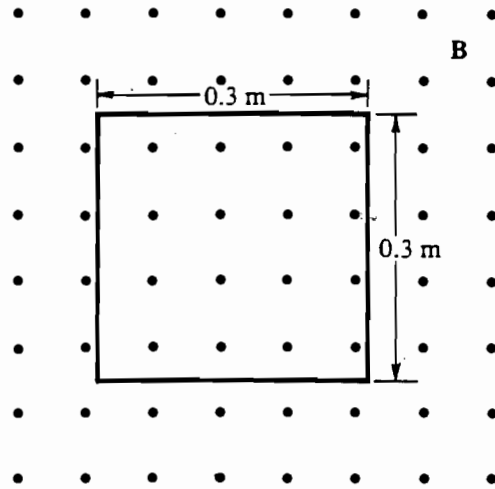
(e) Determine the magnitude of the magnetic force on the loop as a function of time.

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT WORK ON ANY OTHER TEST MATERIALS.



-20-



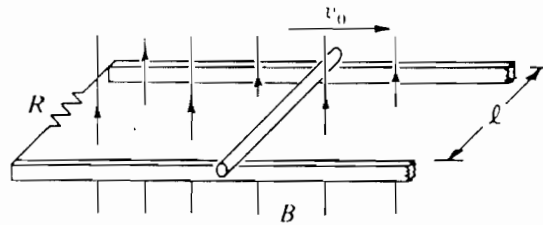
E&M 2. A square wire loop of resistance 6 ohms and side of length 0.3 meter lies in the plane of the page, as shown above. The loop is in a magnetic field  $\mathbf{B}$  that is directed out of the page. At time  $t = 0$ , the field has a strength of 2 teslas; it then decreases according to the equation  $B = 2e^{-4t}$ , where  $B$  is in teslas and  $t$  is in seconds.

- Determine an expression for the flux through the loop as a function of time  $t$  for  $t > 0$ .
- On the diagram above, indicate the direction of the current induced in the loop for time  $t > 0$ .
- Determine an expression for the current induced in the loop for time  $t > 0$ .
- Determine the total energy dissipated as heat during the time from zero to infinity.

GO ON TO THE NEXT PAGE 







- E & M 3. Two horizontal conducting rails are separated by a distance  $\ell$  as shown above. The rails are connected at one end by a resistor of resistance  $R$ . A conducting rod of mass  $m$  can slide without friction along the rails. The rails and the rod have negligible resistance. A uniform magnetic field of magnitude  $B$  is perpendicular to the plane of the rails as shown. The rod is given a push to the right and then allowed to coast. At time  $t = 0$  (immediately after it is pushed) the rod has a speed  $v_0$  to the right.
- Indicate on the diagram above the direction of the induced current in the resistor.
  - In terms of the quantities given, determine the magnitude of the induced current in the resistor at time  $t = 0$ .
  - Indicate on the diagram above the direction of the force on the rod.
  - In terms of the quantities given, determine the magnitude of the force acting on the rod at time  $t = 0$ .

If the rod is allowed to continue to coast, its speed as a function of time will be as follows.

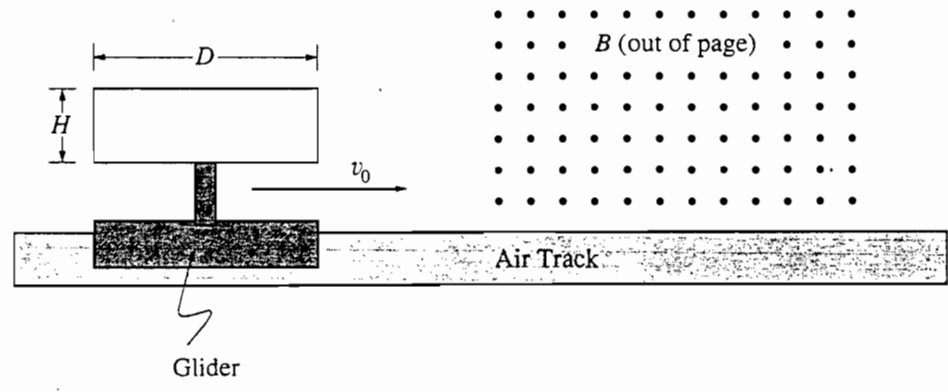
$$v = v_0 e^{-\left(\frac{B^2 \ell^2 t}{Rm}\right)}$$

- In terms of the quantities given, determine the power developed in the resistor as a function of time  $t$ .
- Show that the total energy produced in the resistor is equal to the initial kinetic energy of the bar.

#### END OF SECTION II, ELECTRICITY AND MAGNETISM

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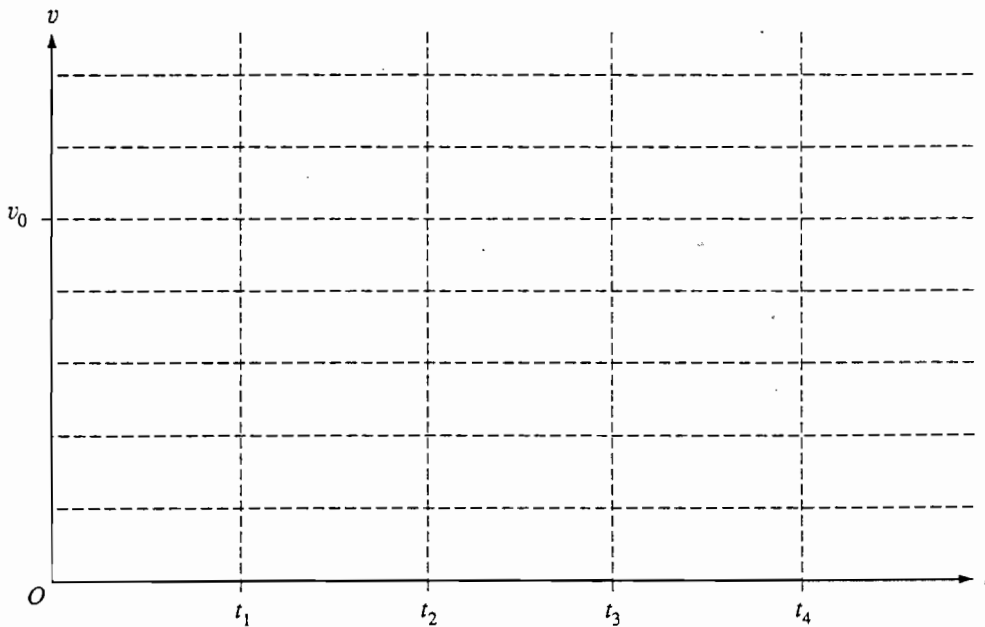


E & M 3. The long, narrow rectangular loop of wire shown above has vertical height  $H$ , length  $D$ , and resistance  $R$ . The loop is mounted on an insulated stand attached to a glider, which moves on a frictionless horizontal air track with an initial speed of  $v_0$  to the right. The loop and glider have a combined mass  $m$ . The loop enters a long, narrow region of uniform magnetic field  $B$  directed out of the page toward the reader. Express your answers to the parts below in terms of  $B$ ,  $D$ ,  $H$ ,  $R$ ,  $m$ , and  $v_0$ .

- (a) What is the magnitude of the initial induced emf in the loop as the front end of the loop begins to enter the region containing the field?
  
- (b) What is the magnitude of the initial induced current in the loop?
  
- (c) State whether the initial induced current in the loop is clockwise or counterclockwise around the loop.

(d) Derive an expression for the velocity of the glider as a function of time  $t$  for the interval after the front edge of the loop has entered the magnetic field but before the rear edge has entered the field.

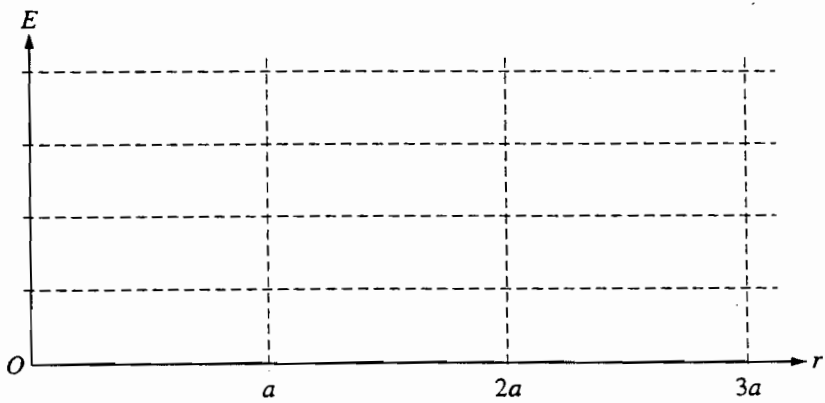
(e) Using the axes below, sketch qualitatively a graph of speed  $v$  versus time  $t$  for the glider. The front end of the loop enters the field at  $t = 0$ . At  $t_1$  the back end has entered and the loop is completely inside the field. At  $t_2$  the loop begins to come out of the field. At  $t_3$  it is completely out of the field. Continue the graph until  $t_4$ , a short time after the loop is completely out of the field. These times may not be shown to scale on the  $t$ -axis below.



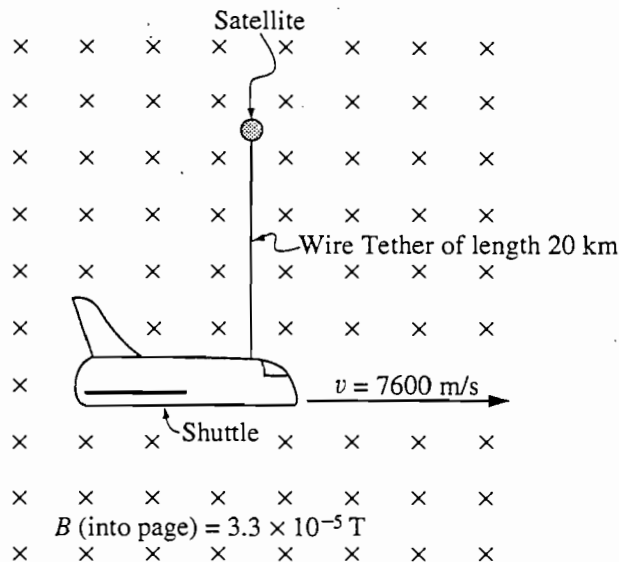


(c) For  $r > a$ , derive an expression for the magnitude  $E$  of the induced electric field in terms of  $r$ ,  $a$ , and  $dB/dt$ .

(d) On the axes below, sketch a graph of  $E$  versus  $r$  for  $0 \leq r \leq 3a$ .



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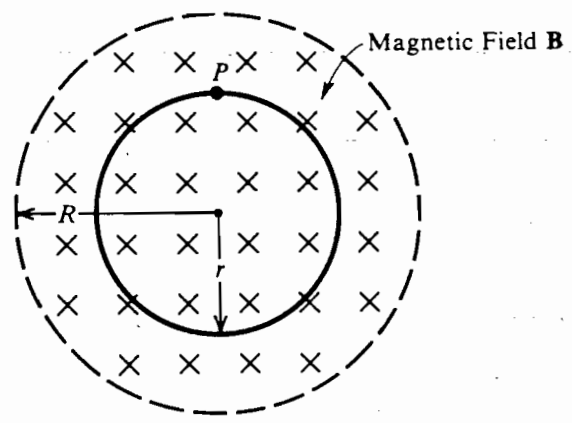
Note: Figure not drawn to scale.

E & M 2. One of the space shuttle missions attempted to perform an experiment in orbit using a tethered satellite. The satellite was to be released and allowed to rise to a height of 20 kilometers above the shuttle. The tether was a 20-kilometer copper-core wire, thin and light, but extremely strong. The shuttle was in an orbit with speed 7,600 meters per second, which carried it through a region where the magnetic field of the Earth had a magnitude of  $3.3 \times 10^{-5}$  tesla. For your calculations, assume that the experiment was completed successfully, that the wire is perpendicular to the magnetic field, and that the field is uniform.

- (a) An emf is generated in the tether.
  - i. Which end of the tether is negative?
  - ii. Calculate the magnitude of the emf generated.

To complete the circuit, electrons are sprayed from the object at the negative end of the tether into the ionosphere and other electrons come from the ionosphere to the object at the positive end.

- (b) If the resistance of the entire circuit is about 10,000 ohms, calculate the current that flows in the tether.
- (c) A magnetic force acts on the wire as soon as the current begins to flow.
  - i. Calculate the magnitude of the force.
  - ii. State the direction of the force.
- (d) By how much would the shuttle's orbital energy change if the current remains constant at the value calculated in (b) for a period of 7 days in orbit?
- (e) Imagine that the astronauts forced a current to flow the other way. What effect would that have, if any, on the orbit of the shuttle? Explain *briefly*.



E&M 3. A spatially uniform magnetic field  $B$ , perpendicular to the plane of the page, exists in a circular region of radius  $R = 0.75$  meter as shown above. A single wire loop of radius  $r = 0.5$  meter is placed concentrically in the magnetic field and in the plane of the page. The magnetic field increases into the page at a constant rate of 60 teslas per second.

- (a) Determine the induced emf in the loop.
- (b) Determine the magnitude and direction of the induced electric field at point  $P$  and indicate its direction on the diagram above.

The wire loop is replaced by an evacuated doughnut-shaped glass tube, within which a single electron orbits at a constant radius  $r = 0.5$  meter when the spatially uniform magnetic field is constant at  $10^{-4}$  tesla.

- (c) Determine the speed of the electron in this orbit.
- (d) The magnetic field is now made to increase at a constant rate of 60 teslas per second as in parts (a) and (b) above. Determine the tangential acceleration of the electron at the instant the field begins to increase.

END OF SECTION II, ELECTRICITY AND MAGNETISM

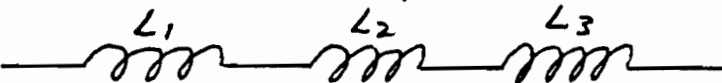
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT WORK ON ANY OTHER TEST MATERIALS.

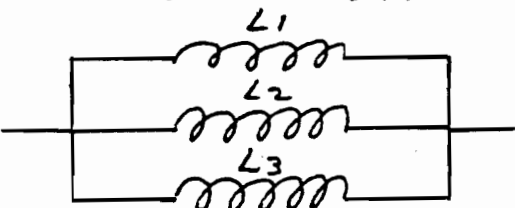
# INDUCTORS

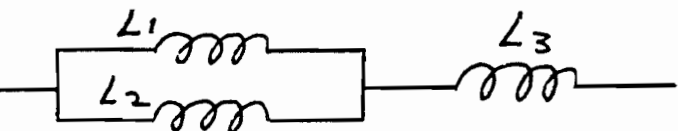
91

1. A COIL HAVING 800 TURNS, A LENGTH OF .2 m AND A CROSS-SECTIONAL AREA OF .025 m<sup>2</sup> IS USED AS AN INDUCTOR. FIND ITS INDUCTANCE.  
(.1 HENRYS)

2. FIND THE TOTAL INDUCTANCE FOR THE FOLLOWING COMBINATIONS.  $L_1 = 50\text{ H}$   $L_2 = 200\text{ H}$   $L_3 = 280\text{ H}$

a)  (530 H)

b)  (35 H)

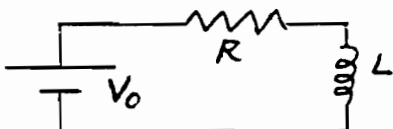
c)  (320 H)

3. AN INDUCTOR HAVING A VOLUME OF .8 m<sup>3</sup> AND AN INDUCTANCE OF 48 H HAS A 5 AMP CURRENT FLOWING THROUGH IT. FIND:

a) THE TOTAL POTENTIAL ENERGY STORED IN ITS MAGNETIC FIELD. (600 J)

b) THE ENERGY DENSITY OF THIS FIELD. (750 J/m<sup>3</sup>)

c) THE MAGNITUDE OF THE MAGNETIC FIELD. (4.34 x 10<sup>-2</sup> T)

4.   $V_0 = 50\text{ V}$   $R = 5\ \Omega$   
 $L = 2\text{ H}$

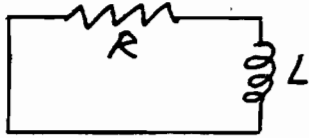
FIND  $V_R$ ,  $i$ ,  $V_L$  AND  $di/dt$  AT EACH OF THE FOLLOWING MOMENTS IN TIME:

a) AT  $t = 0$  (0V, 0A, 50V, 25 AMPS/SEC)

b) AT  $t = \infty$  (50V, 10A, 0V, 0 AMPS/SEC)

c) AT  $t = .4$  SECONDS (31.6V, 6.32A, 18.4V, 9.2 A/S)

5. AFTER AN INFINITE TIME, THE BATTERY IN THE PREVIOUS PROBLEM IS TURNED OFF, LEAVING THE CIRCUIT BELOW.

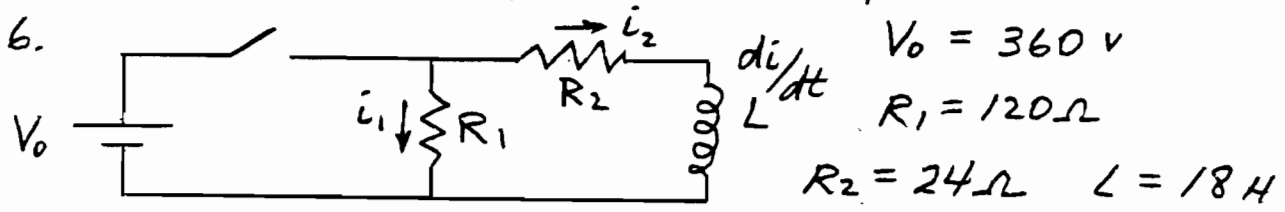


$R = 5\Omega \quad L = 2H$

$i_0 = 10 \text{ AMPS}$

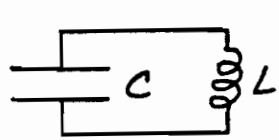
FIND THE FOLLOWING.

- a)  $V_R, V_L$  AND  $di/dt$  AT THE MOMENT  $t=0$  AT WHICH THE BATTERY IS REMOVED. (50V, 50V, 25A/s)
- b) THE TOTAL ENERGY INITIALLY STORED IN THE MAGNETIC FIELD OF THE INDUCTOR. (100 JOULES)
- c)  $i, V_R, V_L$  AND  $di/dt$  AT  $t = .1$  SECOND. (7.79 AMPS, 39V, 39V, 19.47 AMPS/SEC)
- d) THE TIME FOR  $i, V_R$  AND  $V_L$  TO DECREASE TO  $(1/e)^{th}$  OF ITS INITIAL VALUE. (.4 SEC)
- e)  $di/dt$  AT  $t = .4$  SECONDS. (9.2 AMPS/SEC)
- f) THE TOTAL ENERGY DISSIPATED BY THE RESISTOR. (100J)



- a) FIND  $i_1, i_2$  AND  $di/dt$  FOR THE FOLLOWING:
    - i) AT THE INSTANT THE SWITCH IS CLOSED.
    - ii) INFINITELY LONG AFTER THE SWITCH IS CLOSED.
    - iii) AT THE INSTANT THE SWITCH IS REDPENED.
  - b) FIND A FORMULA FOR THE CURRENT IN THE INDUCTOR AS A FUNCTION OF TIME FOR:
    - i) AFTER THE SWITCH IS INITIALLY CLOSED.
    - ii) AFTER THE SWITCH IS REDPENED.
- (3, 0, 20; 3, 15, 0; 15, 15, 120;  $15(1 - e^{-24t/18})$ ;  $15e^{-144t/18}$ )

7. OSCILLATIONS! INITIALLY, THE CAPACITOR <sup>93</sup>  
 HOLDS CHARGE  $Q_0 = 5$  COULOMBS. WHILE THE  
 INDUCTOR CONTAINS NO MAGNETIC FIELD.



$$C = 4 \times 10^{-4} \text{ F}$$

$$L = .0625 \text{ H}$$

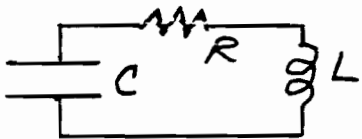
- a) FOR THE INITIAL CONDITIONS, FIND  $V_C$ ,  $V_L$ , P.E.<sub>C</sub>,  
 P.E.<sub>L</sub>, AND  $di/dt$ . (12500, 12500, 31250, 0,  $2 \times 10^5$ )
- b) FOR THE ENSUING OSCILLATIONS, FIND  $\omega$ ,  $f$   
 AND  $T$ . ( $200 \text{ sec}^{-1}$ , 31.83 Hz, .0314 sec)
- c) FIND THE TIME REQUIRED TO NEUTRALIZE THE  
 CAPACITOR. (.00785 sec)
- d) WRITE A FORMULA FOR:  $q$  IN THE CAPACITOR,  
 $|V_C|$ ,  $|V_L|$  AND THE CURRENT IN THE LOOP.

ANSWERS:  $5 \cos 200t$ ,  $12500 \cos 200t$ , " ,  $1000 \sin(200t)$

8. DAMPING!

$$C = 5.9172 \times 10^{-4} \text{ F}$$

$$L = 10 \text{ H}$$



- a) FIND  $R$  SO THAT THE CIRCUIT IS CRITICALLY  
 DAMPED. ( $260 \Omega$ )
- b) WITH THE GIVEN VALUES OF  $C$  AND  $L$ , WE PICK  
 $R = 100 \Omega$  SO AS TO OBSERVE DECAYING OSCILLATIONS.  
 WE START WITH  $Q_0 = .63246 \text{ C}$  AND  $B = 0$ . FIND:
- i)  $\omega$ ,  $f$  AND  $T$  FOR THE E/M OSCILLATIONS.
  - ii)  $\gamma$ , THE DECAY CONSTANT.
  - iii) THE NUMBER OF OSCILLATIONS MADE BY THE  
 CIRCUIT UNTIL  $q$  EQUALS  $(1/e)^{\text{th}}$  OF  $Q_0$ .
  - iv) THE TOTAL ENERGY DISSIPATED BY THE RESISTOR,  
 ( $12 \text{ sec}^{-1}$ , 1.91 Hz, .5236 sec, .2 sec, .382 cycles, 338 J)