SUMMARY OF ELECTROMAGNETISM

I. ELECTRIC FORCE AND ELECTRIC FIELD

A. CHARGE IS ALWAYS CONSERVED.

B. COULOMB'S LAW: $F_E = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$

C. CALCULATION OF $\vec{E}$LECTRIC FIELD

1. POINT CHARGE: $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

   $\vec{E}$ POINTS AWAY FROM ⊕ CHARGES AND TOWARDS ⊖.

2. PARALLEL PLATES: E = CONSTANT

   $E = \sigma/\epsilon_0$ WHERE $\sigma = Q/A$

3. A GROUP OF POINT CHARGES: ADD THE $\vec{E}$ VECTORS ON THE X-AXIS. DO THE SAME FOR THE Y-AXIS. EACH $\vec{E}$ VECTOR IS GIVEN BY $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$ IN THE APPROPRIATE DIRECTION.

4. A CONTINUOUS DISTRIBUTION OF CHARGE:

   $$E = \int dE = \int \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{dQ}{r^2}\right)(\text{TRIGONOMETRIC FUNCTION})$$

   THE TRIGONOMETRIC FUNCTION IS $\cos\theta$, $\sin\theta$, ETC. TO ACCOUNT FOR THE DIRECTION OF $\vec{dE}$. THEN USE $dQ = \lambda\, dX$, $dQ = \lambda\, a\, d\phi$, ETC.

5. SYMMETRIC DISTRIBUTIONS OF CHARGE

   GAUSS' LAW: $\oint \vec{E} \cdot \vec{dS} = q_{inside}/\epsilon_0$

   WHERE $\oint \vec{E} \cdot \vec{dS} = \Phi_E$, THE ELECTRIC FLUX

   AND $\oint dS$ = SURFACE AREA OF THE IMAGINARY SURFACE OVER WHICH THE OBSERVER WALKS.

6. USEFUL $\vec{E}$ FIELDS TO MEMORIZE:

a) ON THE INSIDE OF A CONDUCTOR: $E = 0$.

b) FROM A CONDUCTING PLATE: $E = \sigma / \epsilon_0$

c) NON-CONDUCTING PLATE: $E = \sigma / 2\epsilon_0$

D. WHEN A SMALL CHARGE q IS PUT IN AN $\vec{E}$-FIELD:

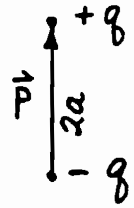
1. $\vec{F}_{ON\ q} = q\vec{E}$

E. WHEN A DIPOLE p IS PUT IN AN $\vec{E}$-FIELD:

1. $\vec{\tau}_{ON\ p} = \vec{p} \times \vec{E}$ $\qquad \vec{\tau}$ = TORQUE [N·m]

2. $U_{OF\ p} = -\vec{p} \cdot \vec{E}$ $\qquad U$ = POTENTIAL ENERGY [J]

WHERE A DIPOLE IS:



$p = 2aq$

POINTING FROM ⊖ TO ⊕

II. ELECTRIC POTENTIAL (V IS ANALOGOUS TO ELEVATION)

A. BASIC DEFINITION: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$

WHERE $d\vec{l}$ IS OUR PATH AS WE MOVE FROM A TO B.

B. SPECIAL CASES:

1. POINT CHARGE: $V = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{r}$

2. PARALLEL PLATES: $V = EX$

C. FOR A POINT CHARGE, $V_\infty = 0$. AS WE MOVE TOWARD A ⊕ CHARGE, V BECOMES A LARGE + NUMBER. TOWARD A ⊖ CHARGE, V BECOMES −.

D. ONCE WE ARE INSIDE A CONDUCTOR, V IS CONSTANT BECAUSE $\vec{E} = 0$ AND NO WORK IS REQUIRED TO MOVE AROUND.

E. CALCULATION OF V:

1. FOR A GROUP OF POINT CHARGES:

$$V = V_1 + V_2 + \ldots = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \ldots\right)$$

WHERE r_i = DISTANCE FROM Q_i TO THE OBSERVER.

V IS A SCALAR, SO DIRECTION DOES NOT MATTER.

2. FOR A CONTINUOUS DISTRIBUTION OF CHARGE:

$$V = \int \frac{1}{4\pi\epsilon_0}\frac{dQ}{r}$$

WHERE $dQ = \lambda\, dx$, ETC.
NO SINES OR COSINES

F. ELECTRIC POTENTIAL ENERGY: $U = qV$

WHERE V IS ANALOGOUS TO ELEVATION AND q IS ANALOGOUS TO A SMALL MASS WHICH WE PUT AT THAT SPOT.

1. $U_{\text{OF } q} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$ NEAR POINT CHARGE Q.

2. $U_{\text{OF } q} = qEx$ BETWEEN PARALLEL PLATES.

G. CHARGE AND SURFACE CHARGE DENSITY

$$V_1 = V_2$$

$$\therefore \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \quad \text{AND} \quad \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

III. CAPACITORS: USED FOR STORING CHARGE

A. $Q = VC$ C ONLY DEPENDS UPON PHYSICAL TRAITS OF THE CAPACITOR

B. SPECIAL CASES: 1. SPHERE: $C = 4\pi\epsilon_0 R$

2. PARALLEL PLATES: $C = \epsilon_0 A/d$ A = AREA, d = SEPARATION

C. IN GENERAL, TO FIND C, USE GAUSS' LAW TO FIND $\vec{E}$. THEN, USE $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{\ell}$ TO FIND V.

FINALLY, USE $Q = VC$ TO FIND C.

D. COMBINATIONS: 1. SERIES: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

2. PARALLEL: $C = C_1 + C_2$

E. ENERGY STORED IN A CAPACITOR:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V \quad [\text{JOULES}]$$

F. ENERGY DENSITY OF ANY ELECTRIC FIELD:

$$u = \frac{1}{2} \epsilon_0 E^2 \quad [\text{JOULES}/m^3]$$

G. FOR DIELECTRICS, THE ONLY FORMULA WHICH IS ALWAYS TRUE: $C_{DIELECTRIC} = K \, C_{VACUUM}$

WHERE K = DIELECTRIC CONSTANT ≥ 1.

IV. CIRCUITS

A. CURRENT DENSITY, $j = i/A \quad \left[\frac{\text{AMPS}}{m^2}\right]$

WHERE i = CURRENT $\quad$ A = CROSS-SECTIONAL AREA OF THE WIRE

B. RESISTIVITY: $\rho = [\text{OHM} \cdot m]$ WHICH ONLY DEPENDS UPON THE TYPE OF MATERIAL.

C. RESISTANCE, $R = \frac{\rho \ell}{A}$ $\quad$ ℓ = LENGTH OF WIRE

A = AREA

D. ∴ OHM'S LAW:

GENERIC FORM $\quad$ FOR A SPECIFIC OBJECT

$E = j\rho \quad\quad V = iR$

E. TEMPERATURE DEPENDENCE: $(\rho - \rho_0) = \rho_0 \alpha (T - T_0)$

$\alpha > 0$ FOR METALS

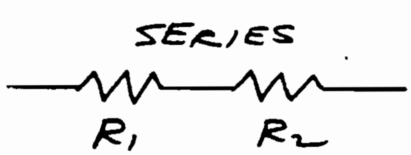
$\alpha < 0$ FOR SEMI-CONDUCTORS $\quad \alpha = \left[\frac{1}{C^\circ}\right]$

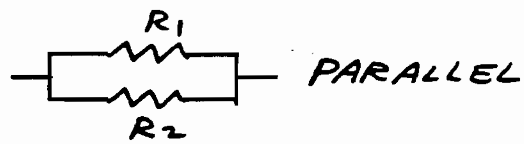
F. POWER IN A CIRCUIT ELEMENT $P = Vi$ [WATTS]

∴ FOR A RESISTOR, $P = i^2 R = V^2/R$

G. RESISTOR COMBINATIONS: 1. SERIES: $R = R_1 + R_2$

2. PARALLEL: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



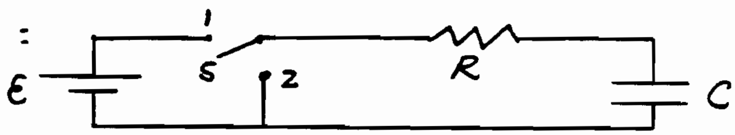


H. KIRCHOFF'S LAWS

1.ST CONSERVATION OF CHARGE : THE CURRENTS FLOWING INTO AN INTERSECTION EQUAL THE CURRENTS FLOWING OUT OF IT.

2.ND CONSERVATION OF ENERGY : THE INCREASES IN POTENTIAL AND THE DECREASES IN POTENTIAL AROUND A CIRCUIT LOOP MUST ADD UP TO ZERO.

I. R-C CIRCUITS :



$\mathcal{E}$ = ELECTROMOTIVE FORCE, BATTERY S = SWITCH

1. TO CHARGE THE CAPACITOR, PUT S TO 1.

$$\mathcal{E} - iR - Q/C = 0 \qquad i = dQ/dt$$

THE SOLUTION TO THIS DIFFERENTIAL EQN IS:

$$Q = \mathcal{E}C\left(1 - e^{-t/RC}\right) \qquad i = \frac{\mathcal{E}}{R}\left(e^{-t/RC}\right)$$

$$\therefore \quad V_C = \mathcal{E}\left(1 - e^{-t/RC}\right) \qquad V_R = \mathcal{E}\, e^{-t/RC}$$

2. TO DISCHARGE THE CAPACITOR, PUT S TO 2.

$$iR + Q/C = 0$$ THE SOLUTION IS :

$$Q = Q_0 e^{-t/RC} \qquad i = -\frac{\mathcal{E}}{R}\, e^{-t/RC}$$

$$V_C = \mathcal{E} e^{-t/RC} \qquad V_R = -\mathcal{E}\, e^{-t/RC}$$

II. UNITS

A. $Q = [\text{COULOMBS}]$

B. $i = [C/\text{SEC} = \text{AMP}]$

C. $E = [N/C = \text{VOLTS}/m]$

D. $V = \left[\text{VOLTS} = \frac{\text{JOULES}}{\text{COULOMB}}\right]$

E. ρ = RESISTIVITY $= [\Omega \cdot m]$
σ = CONDUCTIVITY $= \rho^{-1} = [\Omega^{-1} \cdot m^{-1}]$

F. $\left\{\begin{array}{l} \lambda = [C/m] \\ \sigma = [C/m^2] \\ \rho = [C/m^3] \end{array}\right.$ CHARGE DENSITIES

G. $R = \left[\text{OHMS} = \frac{\text{VOLT}}{\text{AMP}}\right]$

H. $C = \left[\text{FARAD} = \frac{\text{COULOMBS}}{\text{VOLT}}\right]$

VI. THE MAGNETIC FIELD, $\vec{B}$ [TESLA]

A. MAGNETIC FORCE:

1. ON A CHARGE MOVING IN A $\vec{B}$ FIELD:

$$\vec{F}_M = q\,\vec{v} \times \vec{B}$$

$\therefore \quad F_M = q v B \sin\theta$ WITH DIRECTION GIVEN BY THE RIGHT-HAND RULE.

2. ON A WIRE THAT IS CARRYING A CURRENT IN A MAGNETIC FIELD:

$$\vec{F}_M = i\,\vec{\ell} \times \vec{B}$$

B. FOR A SMALL LOOP OF WIRE CARRYING A CURRENT IN A MAGNETIC FIELD:

1. $\vec{\tau} = \vec{\mu} \times \vec{B}$ = THE TORQUE ON LOOP

WHERE μ = MAGNETIC DIPOLE MOMENT

$$\mu = N i A$$

WITH DIRECTION OF $\vec{\mu}$ FROM RIGHT-HAND RULE

2. $U = -\vec{\mu} \cdot \vec{B}$ [JOULES]

C. AMPERE'S LAW: FOR CALCULATING $\vec{B}$ CREATED BY SYMMETRIC CURRENTS.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$$

D. SPECIAL CASES:

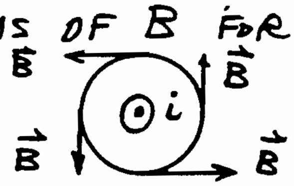
1. FOR A SINGLE WIRE: $B = \dfrac{\mu_0 i}{2\pi r}$

2. FOR A SOLENOID, B IS CONSTANT.

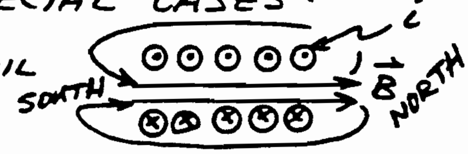
$B = \mu_0 n i$ WHERE $n = \dfrac{\text{\# OF TURNS}}{\text{METER}}$

E. DIAGRAMS OF B FOR THE SPECIAL CASES:

1. WIRE



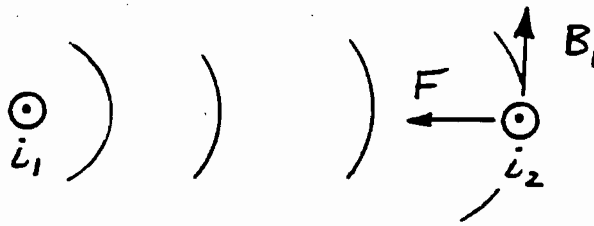
2. COIL



F. FORCE BETWEEN TWO PARALLEL CONDUCTORS

1. LIKE CURRENTS ATTRACT.

2.



$$B_1 = \frac{\mu_0 i_1}{2\pi r}$$

$$F = i_2 \ell B_1 = \frac{\mu_0 \ell i_1 i_2}{2\pi r}$$

ℓ = WIRE LENGTH
r = DISTANCE BETWEEN WIRES

G. BIOT-SAVART LAW: FOR CALCULATING $\vec{B}$ FOR ASYMMETRIC CURRENTS.

$$dB = \frac{\mu_0 i}{4\pi} \frac{d\ell \sin\theta}{r^2}$$

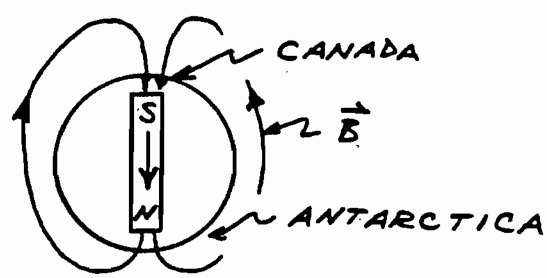
$d\ell$ = SMALL PIECE OF THE WIRE
θ = ANGLE FROM $d\ell$ TO THE OBSERVER.

THEN INTEGRATE OVER THE WHOLE WIRE

$$B = \int dB$$

H. THE EARTH:

$B \simeq 50 \mu T$
$= .5$ GAUSS



VII. INDUCTION

A. LENZ'S LAW GIVES THE DIRECTION OF THE INDUCED CURRENT. THE INDUCED CURRENT MOVES SO THAT ITS MAGNETIC FIELD OPPOSES THE CHANGE IN THE EXTERNAL $\vec{B}$ FIELD.

B. FARADAY'S LAW GIVES THE MAGNITUDE OF THE INDUCED VOLTAGE. $V_{INDUCED} = \oint \vec{E} \cdot \vec{d\ell} = -\frac{d\Phi_B}{dt}$

WHERE Φ_B = MAGNETIC FLUX

$$= \oint \vec{B} \cdot \vec{dS} \quad [\text{TESLA} \cdot m^2 = \text{WEBER}]$$

* C. TO WORK MOST PROBLEMS: USUALLY, EITHER THE EXTERNAL MAGNETIC FIELD IS CHANGING OR THE AREA IS CHANGING, NOT BOTH.

IN THAT CASE, FARADAY'S LAW BECOMES:

1. $$V_{INDUCED} = -A \frac{dB}{dt}$$

OR 2. $$V_{INDUCED} = -B \frac{dA}{dt}$$

THE MINUS SIGN IS OF NO CONCERN. LENZ'S LAW TAKES CARE OF DIRECTION.

VIII. CONCLUSION: MAXWELL'S EQUATIONS

A. CHARGES PRODUCE ELECTRIC FIELDS

$$\oint \vec{E} \cdot \vec{dS} = q_{INSIDE} / \epsilon_0$$

B. MAGNETIC MONOPOLES DO NOT EXIST.

$$\oint \vec{B} \cdot \vec{dS} = 0$$

C. CHANGING MAGNETIC FIELDS PRODUCE ELECTRIC FIELDS. $$\oint \vec{E} \cdot \vec{d\ell} = -\frac{d\Phi_B}{dt}$$

D. CURRENTS AND CHANGING ELECTRIC FIELDS PRODUCE MAGNETIC FIELDS.

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 1997

MECHANICS

$v = v_0 + at$

$s = s_0 + v_0 t + \frac{1}{2} at^2$

$v^2 = v_0^2 + 2a(s - s_0)$

$\Sigma \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\, dt = \triangle \mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_f \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{s}$

$K = \frac{1}{2} m v^2$

$P = \frac{dW}{dt}$

$\triangle U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\Sigma \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \Sigma m r^2$

$\mathbf{r}_{cm} = \Sigma m\mathbf{r} / \Sigma m$

$v = r\omega$

$\mathbf{L} = I\boldsymbol{\omega}$

$K = \frac{1}{2} I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2} kx^2$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$F_G = -\frac{Gm_1m_2}{r^2}$

$U_G = -\frac{Gm_1m_2}{r}$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = distance
s = displacement
T = period
t = time
U = potential energy
v = velocity or speed
W = work
x = displacement
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\int \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum \frac{q}{r}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2} QV = \frac{1}{2} CV^2$

$R = \frac{\rho\ell}{A}$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$\mathbf{F} = \int I\, d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} LI^2$

A = area
B = magnetic field strength
C = capacitance
d = distance
E = electric field strength
$\mathcal{E}$ = emf
F = force
I = current
L = inductance
ℓ = length
n = number of loops of wire per unit length
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ = magnetic flux
κ = dielectric constant