SUMMARY OF CLASSICAL MECHANICS

I. KINEMATICS

A. VARIABLES: POSITION $\vec{r} = (x, y) = x\hat{i} + y\hat{j}$

VELOCITY $\vec{v} = (v_x, v_y) = v_x\hat{i} + v_y\hat{j}$

ACCELERATION $\vec{a} = (a_x, a_y) = a_x\hat{i} + a_y\hat{j}$

B. BASIC RELATIONSHIPS WHICH ALWAYS HOLD:

1. $\vec{v} = \dfrac{d\vec{r}}{dt}$

2. $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

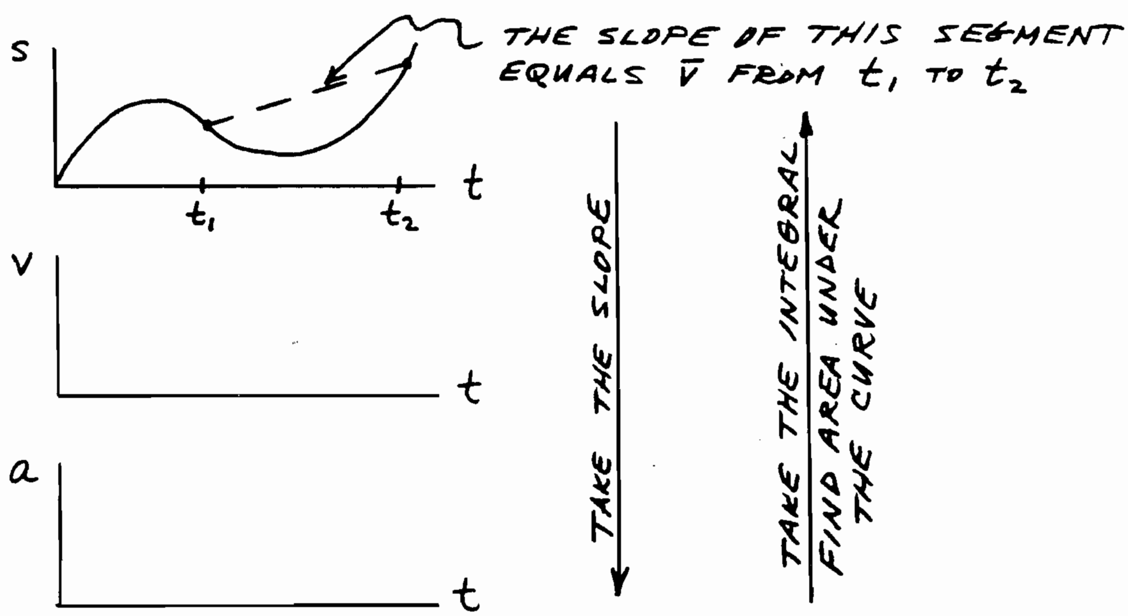
3. $\vec{r} = \int \vec{v}\,dt$

4. $\vec{v} = \int \vec{a}\,dt$

5. $$\bar{v} = \frac{\text{TOTAL DISPLACEMENT}}{\text{TOTAL TIME}} = \frac{\vec{r} - \vec{r}_0}{t}$$

$$\bar{v} = \left(\frac{1}{t - t_0}\right) \int_{t_0}^{t} \vec{v}\,dt$$

C. GRAPHICAL RELATIONSHIPS



D. IF $\vec{a}$ IS CONSTANT IN MAGNITUDE AND DIRECTION:

$$s = \frac{1}{2}at^2 + V_0 t + s_0 \qquad s - s_0 = \bar{V}t = \left(\frac{V+V_0}{2}\right)t$$

$$V = at + V_0 \qquad V^2 - V_0^2 = 2a(s - s_0)$$

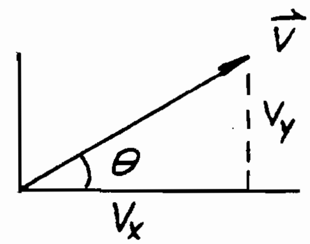
E. X AND Y MOTIONS ARE INDEPENDENT OF EACH OTHER. TO FIND THE COMPONENTS AND TO RECONSTRUCT A VECTOR:

$$V_x = V\cos\theta$$

$$V_y = V\sin\theta$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$



II. DYNAMICS

A. LAW OF INERTIA: $\vec{V}$ IS CONSTANT, IN BOTH SPEED AND DIRECTION, UNLESS THERE IS AN UNBALANCED, EXTERNAL FORCE.

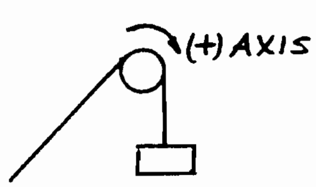
B. FORCES CAUSE ACCELERATION: $m\vec{a} = \Sigma \vec{F}_{EXTERNAL}$

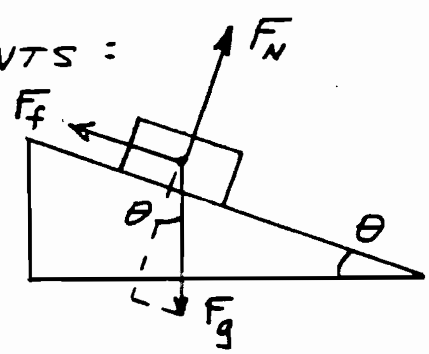
$$F_g = mg \qquad F_f = \mu F_N$$

BEWARE: SOLVE FOR F_N FROM THE Y-AXIS EQUATION. F_N IS NOT NECESSARILY EQUAL TO F_g.

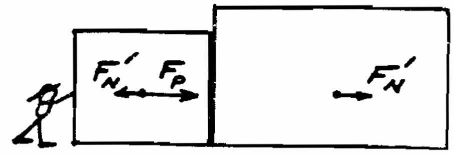
C. ACTION/REACTION: IF BODY A EXERTS A FORCE ON BODY B, BODY B EXERTS AN EQUAL, OPPOSITE FORCE ON BODY A.

D. PROBLEM SOLVING HINTS:





FOR MULTIPLE OBJECTS, WRITE NEWTON'S LAW FOR EACH OBJECT USING ONLY THOSE FORCES WHICH DIRECTLY ACT ON THAT OBJECT. SOLVE THE SIMULTANEOUS EQUATIONS.

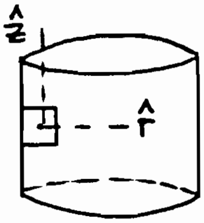


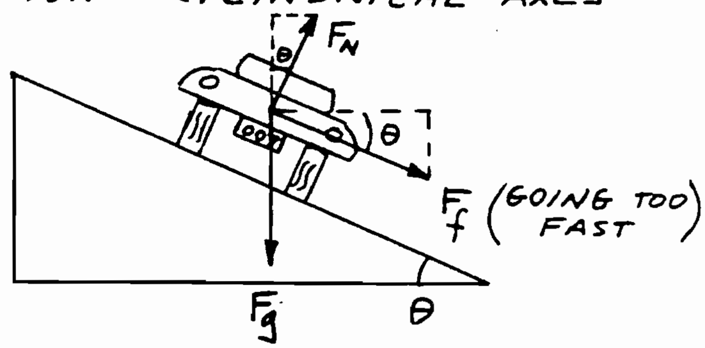
F_N' ARE EQUAL / OPPOSITE

$$m a = F_{PUSH} - F_N'$$

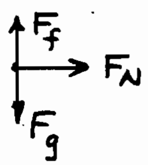
$$M a = F_N'$$

III. UNIFORM CIRCULAR MOTION : CYLINDRICAL AXES

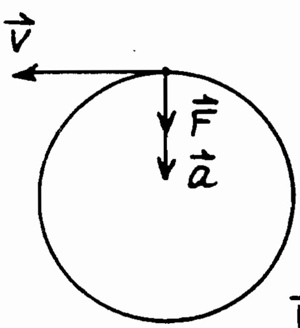




FORCE DIAGRAM



r-AXIS	z-AXIS
$m\left(\frac{v^2}{r}\right) = \Sigma F_r$	$m a_z = \Sigma F_z$ (MOSTLY, $a_z = 0$)



$\vec{a}$ AND $\vec{F}$ ARE ALWAYS PARALLEL.

WHEN $\vec{F}$ AND $\vec{v}$ ARE PERPENDICULAR, ONLY THE DIRECTION CHANGES.

$\vec{v}, \vec{a}, \vec{F}$ ARE ALL NOT CONSTANT, DIRECTION IS CHANGING.

$v = \omega r$ $\quad$ ω = ANGULAR VELOCITY [RADIANS/SEC]

$f = \omega / 2\pi$ $\quad$ f = FREQUENCY [Hz]

IV. WORK AND ENERGY

A. BASIC RELATIONSHIPS

$$\text{WORK} = \int \vec{F} \cdot d\vec{x} = \vec{F} \cdot \vec{x}$$

IF $\vec{F}$ IS CONSTANT, UNLIKE SPRINGS

$$POWER = \frac{WORK}{TIME} \qquad KE = \frac{1}{2} m v^2$$

$$F_g = mg \qquad F_s = -kx$$
$$PE_g = mgh \qquad PE_s = \frac{1}{2} k x^2$$

* B. WORK - ENERGY THEOREM : THE TOTAL WORK DONE BY ALL THE FORCES ACTING ON AN OBJECT EQUALS THE CHANGE IN KINETIC ENERGY.

$$KE - KE_0 = \sum Work = \int F\,dx \qquad (1)$$

FOR EXAMPLE, IN UNIFORM CIRCULAR MOTION, ALL FORCES ARE PERPENDICULAR TO THE MOTION, THUS THEY DO NO WORK. THEREFORE, THE K.E. DOES NOT CHANGE AND THE PARTICLE REMAINS AT THE SAME SPEED. IT ONLY CHANGES ITS DIRECTION.

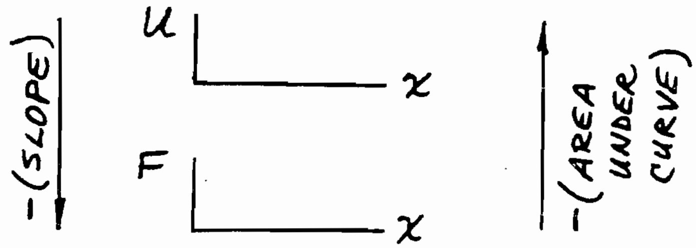
C. CONSEQUENCE: THE WORK DONE BY CONSERVATIVE FORCES CAN BE CALCULATED ONCE IN A LIFETIME AND MADE INTO P.E. FORMULAS. EQUATION (1) BECOMES: $KE_0 + PE_0 + PE_0' \pm Work = KE + PE + PE'$ WHERE "$\pm$ WORK" IS THAT WORK DONE BY NON-CONSERVATIVE FORCES, WHICH WILL NOT HAVE P.E. ASSOCIATE WITH THEM.

D. $F = -\frac{dU}{dx}$ $\qquad U - U_0 = -\int F\,dx$

$= -$(WORK DONE BY F)

$=$ (WORK DONE BY YOU AGAINST F)

E. GRAPHS:



F. USEFUL RELATIONSHIPS FOR POWER

1. IF ONLY THE K.E. IS CHANGING:

$$P = \frac{\Delta KE}{\Delta t} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt} = mva$$

2. IF F IS GIVEN: $P = \frac{\vec{F} \cdot \vec{x}}{t} = \vec{F} \cdot \vec{v}$

3. IF ONLY P.E. IS CHANGING: $P = \frac{\Delta PE}{\Delta t} = \frac{mgh}{t}$

V. LINEAR MOMENTUM

A. $Mx_{cm} = m_1 x_1 + m_2 x_2$ FINDS THE CENTER OF MASS

THE C.M. CAN MOVE ONLY IF THERE ARE EXTERNAL FORCES.

B. $\vec{p} = m\vec{v}$ $\qquad p - p_0 = \int F\,dt = J =$ IMPULSE

$$\frac{d\vec{p}}{dt} = \Sigma F_{ext} \qquad \frac{p - p_0}{t} = \Sigma F_{EXTERNAL}$$

C. COLLISIONS: $\vec{p}_0 = \vec{p}_F$

FOR MULTI-DIMENSIONAL COLLISIONS, BEGIN BY WRITING $p_0 = p_F$ FOR EACH AXIS.

D. NOTE, MOMENTUM IS CONSERVED WHEN THERE IS NO NET, EXTERNAL FORCE.

VI. ROTATIONAL KINEMATICS

A. BASIC RELATIONSHIPS WHICH ALWAYS HOLD:

1. $\omega = \dfrac{d\theta}{dt}$

3. $\theta = \int \omega \, dt$

2. $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

4. $\omega = \int \alpha \, dt$

θ IS NOT A VECTOR SINCE IT DOES NOT COMMUTE.

$\vec{\omega}$ AND $\vec{\alpha}_{\text{TANGENTIAL}}$ ARE PARALLEL, GIVEN BY THE RIGHT HAND CURL RULE.

B. ROTATIONAL $\rightleftarrows$ LINEAR

$$s = r\theta \qquad\qquad a_{\text{RADIAL}} = \frac{v^2}{R}$$

$$v = r\omega$$

$$a_T = r\alpha \qquad a_T = \text{TANGENTIAL ACCELERATION}$$

TO DETERMINE DIRECTIONS: $\begin{cases} \vec{v} = \vec{\omega} \times \vec{r} \\ \vec{a}_T = \vec{\alpha} \times \vec{r} \\ \vec{a}_r = \vec{\omega} \times \vec{v} \end{cases}$

C. FOR CONSTANT ANGULAR ACCELERATION

$$\omega = \alpha t + \omega_0 \qquad\qquad \theta = \tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$$

$$\theta - \theta_0 = \bar{\omega} t = \left(\frac{\omega + \omega_0}{2}\right) t \qquad\qquad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

VII. ROTATIONAL DYNAMICS

A. BASIC RELATIONSHIPS

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad\qquad \tau = rF\sin\theta = r_\perp F$$

$$\vec{\ell} = \vec{r} \times \vec{p} \qquad\qquad \ell = rp\sin\theta = r_\perp p$$

B. SUMMARY

Table 12-2

Rectilinear Motion		Rotation about a Fixed Axis	
Displacement	x	Angular displacement	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass (translational inertia)	M	Rotational inertia	I
Force	$F = Ma$	Torque	$\tau = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Linear momentum	Mv	Angular momentum	$I\omega$

C. $KE_{o\,TRANSLATION} + KE_{o\,ROTATION} + PE_o \pm WORK = KE_T + KE_R + PE$

1. FOR ROLLING WITHOUT SLIPPING: $V = r\omega$
2. IF THE OBJECT IS SLIPPING, A F_f WILL EXERT A TORQUE TO REDUCE THE ROTATIONAL SPEED.

THUS: $\tau = I\alpha$

$$-F_f\, r = I\left(\frac{v_F - v_o}{t}\right)$$

FOR DECELERATION

WHERE $v_F = r\omega$

D. IN THE ABSENCE OF EXTERNAL TORQUES: $\ell_o = \ell$, IE, ANGULAR MOMENTUM IS CONSERVED.

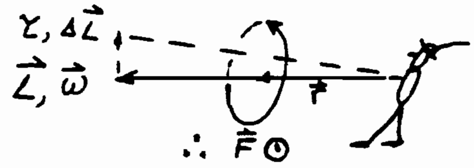
$$I_o\omega_o = I\omega$$

MECHANICAL ENERGY IS NOT CONSERVED.

$$\frac{1}{2}I_o\omega_o^2 \pm WORK = \frac{1}{2}I\omega^2$$

WHERE WORK IS SUPPLIED BY ATHLETE'S MUSCLES.

E. BICYCLE WHEELS



THE PHYSICIST PUSHES THE WHEEL UPWARD.

$\Delta L / \Delta t = \tau_{EXT}$, SUPPLIED BY HAND

$$\vec{\tau} = \vec{r} \times \vec{F}_{HAND}$$

$\therefore$ HAND MUST PUSH OUT OF PAPER OR WHEEL WILL GO INTO THE PAPER.

F. PARALLEL-AXIS THEOREM: $I = I_{cm} + mh^2$

VIII. EQUILIBRIUM OF SOLID OBJECTS

RECIPE: DRAW A FORCE DIAGRAM. PUT AN AXIS ON A CONVENIENT SPOT.

X-AXIS	Y-AXIS	ROTATIONS
$0 = \Sigma F_x$	$0 = \Sigma F_y$	$0 = \Sigma \tau = \Sigma r_\perp F$

IX. SIMPLE HARMONIC MOTION

A. S.H.M. OCCURS WHEN THE RESTORING FORCE IS PROPORTIONAL TO THE POSITION.

B. EXAMPLE: SPRINGS FOLLOWING HOOKE'S LAW:

$$m\ddot{x} = -kx$$

SOLUTION: $x = A\cos(\omega t + \phi)$ $\qquad \omega = \sqrt{k/m}$

$v = -A\omega \sin(\omega t + \phi)$ $\qquad T = 2\pi\sqrt{m/k}$

$A = -A\omega^2 \cos(\omega t + \phi)$

C. ENERGY CONSIDERATIONS

$$KE + PE = \tfrac{1}{2} m\left[-A\omega\sin(\omega t+\phi)\right]^2 + \tfrac{1}{2}k\left[A\cos(\omega t+\phi)\right]^2$$
$$= \tfrac{1}{2}kA^2$$

D. HINTS: $\phi = 0$ IF THE MASS STARTS FROM REST.
k IS INVERSELY PROPORTIONAL TO THE NUMBER OF COILS IN A SPRING.

E. PENDULUMS:

1. SIMPLE $\qquad m\ddot{x} = -\left(mg/\ell\right)x$

$$\therefore\ \omega = \sqrt{g/\ell} \qquad T = 2\pi\sqrt{\ell/g}$$

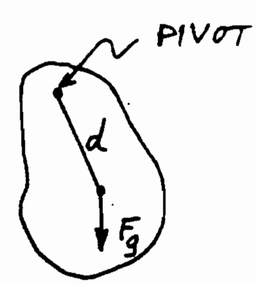
2. TORSIONAL $\qquad I\ddot{\theta} = -k\theta$

$$\therefore\ \omega = \sqrt{k/I} \qquad T = 2\pi\sqrt{I/k}$$

WHERE I = MOMENT OF INERTIA
k = TORSIONAL CONSTANT OF WIRE

3. PHYSICAL: $I\ddot{\theta} = -(mgd)\theta$

$$\therefore\ \omega = \sqrt{\frac{mgd}{I}}$$



X. GRAVITATION

A. NEWTON'S LAW OF UNIVERSAL GRAVITATION

$$F_g = \frac{GmM}{r^2}$$

B. ACCELERATION DUE TO GRAVITY

$$g = \frac{GM}{r^2}$$

r = DISTANCE TO CENTER OF PLANET

C. GRAVITATIONAL POTENTIAL ENERGY

1. NEAR SURFACE OF PLANET — WHERE "g" CAN BE ASSUMED TO BE CONSTANT

$$U = mgh$$

2. IN GENERAL, $U_\infty = 0$ AND

U = − WORK TO MOVE PARTICLE FROM INFINITY

$$U = -\frac{GmM}{r}$$

3. FOR A MANY PARTICLE SYSTEM:

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)$$

D. ESCAPE VELOCITY

$$KE_0 + PE_0 = KE + PE$$

$$\frac{1}{2}mv_0^2 - \frac{GmM}{R} = 0 + 0$$

E. SATELLITES:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{GmM}{r}\right)$$

$$U = -GmM/r$$

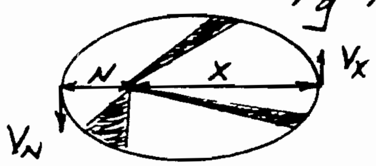
$$E_{TOTAL} = -\frac{GmM}{2r}$$

F. KEPER'S LAWS

1^{ST}: LAW OF EQUAL AREAS

CONSERVATION OF ANGULAR MOMENTUM DUE TO ABSENCE OF EXTERNAL TORQUES.

F_g IS CENTRALLY DIRECTED TOWARD SUN



$$L_N = L_X$$
$$P_N N = P_X X$$
$$m V_N N = m V_X X$$

$$\frac{V_X}{V_N} = \frac{N}{X}$$

2^{ND}: LAW OF ELLIPTICAL ORBITS

AS A RESULT OF INVERSE SQUARE LAW

3^{RD}: LAW OF PERIODS

$$\left(\frac{T^2}{R^3}\right)_A = \left(\frac{T^2}{R^3}\right)_B$$

WHERE BOTH A AND B ORBIT THE SAME OBJECT.

R = AVERAGE DISTANCE FROM CENTER OF SUN TO CENTER OF PLANET

= SEMI-MAJOR AXIS = $(X+N)/2$

DERIVATION: $ma = F_g$

$$\frac{mV^2}{R} = \frac{GmM}{R^2} \qquad V = \frac{2\pi R}{T}$$

$$\frac{m\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{GmM}{R^2}$$

$$\left(\frac{T^2}{R^3}\right) = \left(\frac{4\pi^2}{GM}\right), \text{ A CONSTANT}$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 1997

MECHANICS

$v = v_0 + at$

$s = s_0 + v_0 t + \frac{1}{2} at^2$

$v^2 = v_0^2 + 2a(s - s_0)$

$\Sigma \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\, dt = \triangle \mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_f \leq \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{s}$

$K = \frac{1}{2} m v^2$

$P = \frac{dW}{dt}$

$\triangle U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\Sigma \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \Sigma m r^2$

$\mathbf{r}_{cm} = \Sigma m\mathbf{r} / \Sigma m$

$v = r\omega$

$\mathbf{L} = I\boldsymbol{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2} kx^2$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$F_G = -\frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = distance
s = displacement
T = period
t = time
U = potential energy
v = velocity or speed
W = work
x = displacement
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\int \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0} \Sigma \frac{q}{r}$

$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa \epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2} QV = \frac{1}{2} CV^2$

$R = \frac{\rho \ell}{A}$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$\mathbf{F} = \int I\, d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 n I$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L \frac{dI}{dt}$

$U_L = \frac{1}{2} LI^2$

A = area
B = magnetic field strength
C = capacitance
d = distance
E = electric field strength
$\mathcal{E}$ = emf
F = force
I = current
L = inductance
ℓ = length
n = number of loops of wire per unit length
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ = magnetic flux
κ = dielectric constant