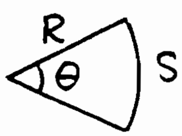
UNIT CONVERSIONS

1. CONVERT $9.8\ m/s^2$ INTO MPH/SEC. NOTE THAT ONE MILE EQUALS 1610 m. (21.9 MPH/SEC)

2. IN VACUUM, LIGHT TRAVELS AT $3 \times 10^8\ m/s$. FIND THE DISTANCE TRAVELED BY LIGHT IN ONE YEAR. THIS DISTANCE IS CALLED A LIGHT-YEAR. (9.467×10^{15} m OR 5.88×10^{12} miles)

3. THE MEAN DISTANCE BETWEEN THE EARTH AND THE SUN IS 149.6×10^6 km, WHICH IS CALLED ONE ASTRONOMICAL UNIT. FIND THE RADII OF THE FOLLOWING ORBITS IN A.U..
   a) THE MOON, WHICH IS 3.8×10^5 km FROM EARTH. (.00254)
   b) NEPTUNE, WHICH IS 4496.6×10^6 km FROM SUN. (30.06)

4. A PARSEC IS THE DISTANCE AT WHICH ONE ASTRONOMICAL UNIT SUBTENDS AN ANGLE OF ONE SECOND. RECALL THAT



TO CALCULATE ARC LENGTH, $S = R\theta$, WHERE θ IS IN RADIANS. NOW, $S = 149.6 \times 10^6$ km.
RECALL: 2π RADIANS $= 360°$
$1° = 60$ MINUTES OF ARC, $1' = 60''$ OF ARC.

FIND THE DISTANCE R, WHICH IS ONE PARSEC, IN KILOMETERS, IN A.U. AND IN LIGHT-YEARS. (3.086×10^{13} km, 2.06×10^5 A.U., 3.26 LIGHT-YEARS)

5. AT A SYMPOSIUM IN 415 B.C. SOCRATES APPEASED THE GODS BY POURING A LIBATION OF WINE ONTO AN ALTER. THE LIBATION CONTAINED 60g OF WATER, WHICH ARE NOW PERFECTLY MIXED WITH ALL THE WATER IN THE EARTH'S OCEANS, RIVERS, LAKES AND CLOUDS, A TOTAL OF 1×10^{21} KG. MR. HARVIE HAS 40 KG OF WATER IN HIS BODY.
   A) USE MENDELEEV'S CHART AND AVOGADGRO'S NUMBER TO FIND THE NUMBER OF WATER MOLECULES IN MR. H'S BODY. (1.34×10^{27} molecules)
   B) FIND THE NUMBER OF WATER MOLECULES FROM SOCRATES' LIBATION WHICH ARE IN MR. H. (80267)

6. THE RADIUS OF A NUCLEUS IS GIVEN BY : $R = 1.2 \times 10^{-15} A^{1/3}$
WHERE A = THE ATOMIC MASS NUMBER AND
R = THE RADIUS IN [METERS].

A) FIND THE RADIUS OF AN IRON-56 NUCLEUS.

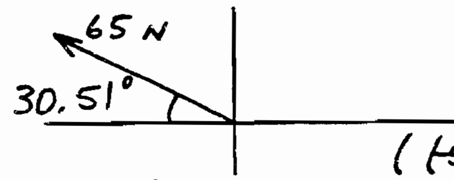
B) IRON-56 HAS A NUCLEAR MASS OF 9.28×10^{-26} KG. FIND THE DENSITY OF THIS NUCLEUS.

C) OUR SUN HAS A MASS OF 2×10^{30} KG. UPON COMPLETION OF ITS FUSION REACTION, ITS GRAVITY WOULD COLLAPSE THE STAR FORCING THE ELECTRONS INTO THE NUCLEUS, NEUTRALIZING THE PROTONS. THIS NEUTRON STAR WOULD HAVE THE DENSITY OF ATOMIC NUCLEI. FIND THE RADIUS OF THE NEUTRON STAR RESULTING FROM THE GRAVITATIONAL COLLAPSE OF OUR SUN.

(4.59×10^{-15} m, 2.29×10^{17} KG/m^3, 12775 m = 12.8 km)

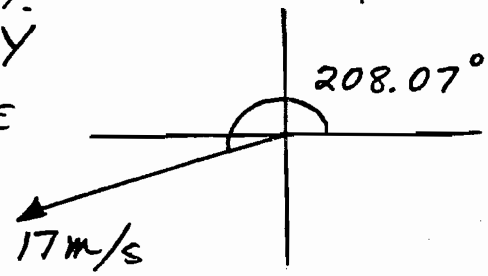
VECTOR ANALYSIS

1. FIND THE X AND Y COMPONENTS FOR THE VECTOR AT THE RIGHT.



((-56, 33) N)

2. FIND THE X AND Y COMPONENTS FOR THE VECTOR DRAWN TO THE RIGHT.



((-15, -8))

3. A DISPLACEMENT VECTOR, $\vec{r}$, IS SPECIFIED BY $(x, y) = (5, -12)$. SKETCH A DIAGRAM. DETERMINE ITS MAGNITUDE AND DIRECTION.

(13 m AT 67.3° IN FOURTH QUADRANT, OR 292.6°)

4. A JET, WHOSE VELOCITY, $\vec{V}_J$, EQUALS $(30, 72)$, ENCOUNTERS A WIND, WHOSE VELOCITY $\vec{V}_W$ EQUALS $(16, -20)$. THE RESULTING VELOCITY OF THE JET IS $\vec{V}_R$, WHERE $\vec{V}_R = \vec{V}_J + \vec{V}_W$. FIND:

A) A VECTOR EXPRESSION FOR THE RESULTING VELOCITY. ((46, 52))

B) THE MAGNITUDE AND DIRECTION OF $\vec{V}_R$.

(69.4 m/s AT 48.5°, FIRST QUADRANT)

C) SKETCH A DIAGRAM OF THE THREE VECTORS.

5. A JET, WHOSE RESULTING VELOCITY $\vec{V}_R$ IS $(-120, -160)$, FLIES WHILE SUBJECT TO A WIND, WHOSE VELOCITY $\vec{V}_W$ IS $(30, -40)$. $\vec{V}_R - \vec{V}_W = \vec{V}_J$.

FIND: A) A VECTOR EXPRESSION FOR $\vec{V}_J$. ((-150, -120))

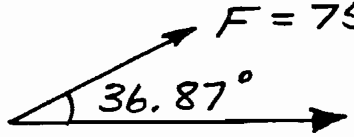
B) THE MAGNITUDE AND DIRECTION OF $\vec{V}_J$.

(192 m/s AT 38.6° INTO THIRD QUADRANT OR 218.6°)

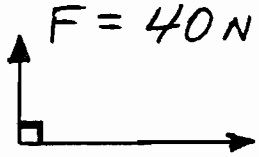
C) SKETCH A DIAGRAM.

6. WORK, $W = \vec{F} \cdot \vec{d}$ WHERE $\vec{F}$ = FORCE AND $\vec{d}$ = DISPLACEMENT. USE $W = Fd\cos\theta$ TO FIND THE WORK DONE IN EACH OF THE FOLLOWING CASES.

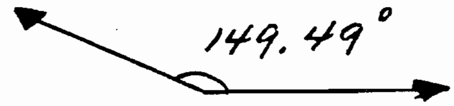
a) $F = 75N$, $36.87°$, $d = 8m$



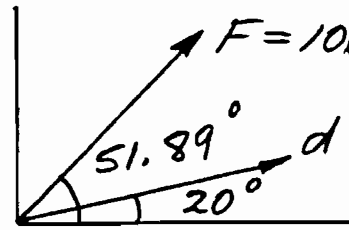
b) $F = 40N$, $d = 8m$



c) $F = 65N$, $149.49°$, $d = 8m$



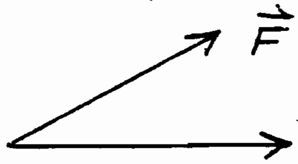
d) $F = 106N$, $51.89°$, $20°$, $d = 5m$



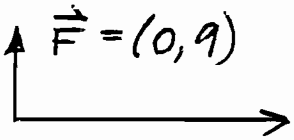
(480, 0, −448, 450 Nm)

7. WORK, $W = \vec{F} \cdot \vec{d}$. USE THE INNER PRODUCT TO FIND THE WORK DONE IN EACH CASE.

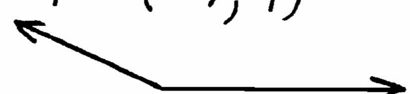
a) $\vec{F} = (7,9)$, $\vec{d} = (8,0)$



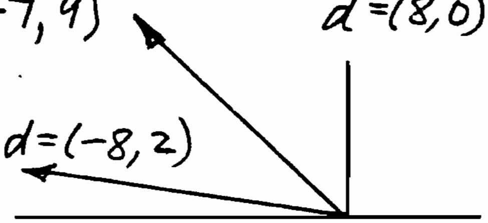
b) $\vec{F} = (0,9)$, $\vec{d} = (8,0)$



c) $\vec{F} = (-7,9)$, $\vec{d} = (8,0)$



d) $\vec{F} = (-7,9)$, $d = (-8,2)$



(56, 0, −56, 74 Nm)

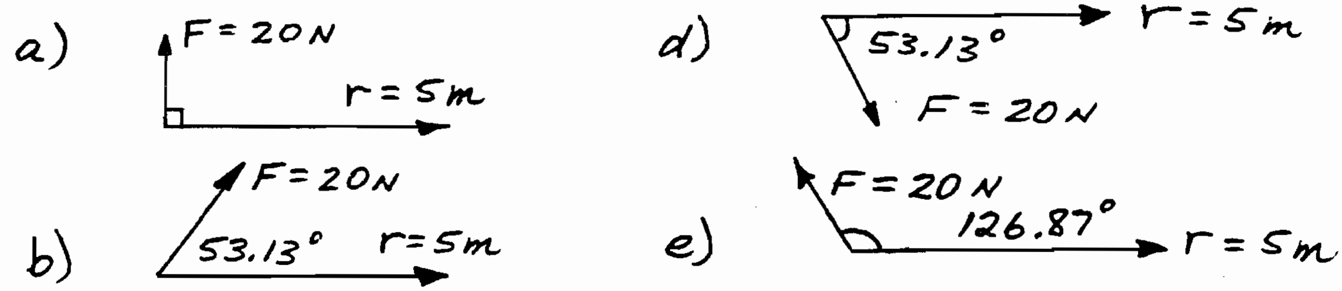
8. GIVEN: $\vec{d} = (-15, 20, 60)$ AND $\vec{F} = (9, 12, 8)$

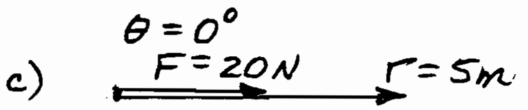
a) USE THE INNER PRODUCT TO FIND THE WORK DONE BY THE FORCE. (585 Nm)

b) FIND THE MAGNITUDE OF $\vec{d}$ AND OF $\vec{F}$.
(65 m, 17 N)

c) FIND THE ANGLE BETWEEN $\vec{d}$ AND $\vec{F}$.
(58°)

9. TORQUE, $\vec{\tau} = \vec{r} \times \vec{F}$ WHERE $\vec{r}$ = RADIUS AND $\vec{F}$ = FORCE. USE $\tau = rF\sin\theta$ TO FIND THE MAGNITUDE OF THE TORQUE IN EACH OF THE FOLLOWING CASES.

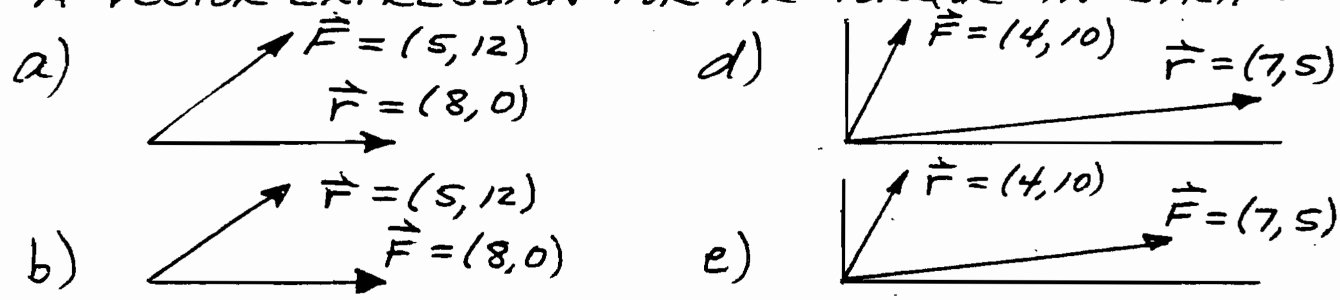


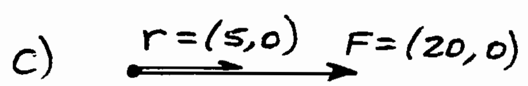


(100, 80, 0, 80, 80 Nm)

10. TORQUE, $\vec{\tau} = \vec{r} \times \vec{F}$. USE THE RIGHT HAND RULE TO FIND THE DIRECTION OF $\vec{\tau}$ FOR THE CASES IN PROBLEM 9. ($\odot$, $\odot$, NONE, $\otimes$, $\odot$)

11. $\vec{\tau} = \vec{r} \times \vec{F}$. USE THE OUTER PRODUCT TO FIND A VECTOR EXPRESSION FOR THE $\vec{\text{TORQUE}}$ IN EACH:





$\left(\begin{matrix} (0,0,96), (0,0,-96), (0,0,0) \\ (0,0,50), (0,0,-50) \end{matrix} \right)$

12. $\vec{\tau} = \vec{r} \times \vec{F}$. GIVEN: $\vec{r} = (-15, 20, 60)$ AND $\vec{F} = (9, 12, 8)$.

a) USE THE OUTER PRODUCT TO FIND A VECTOR EXPRESSION FOR $\vec{\tau}$. ((-560, 660, -360))

b) FIND THE MAGNITUDES OF $\vec{r}$, $\vec{F}$ AND $\vec{\tau}$.
(65 m, 17N, 937.44 Nm)

c) FIND THE ANGLE BETWEEN $\vec{r}$ AND $\vec{F}$. (58°)

13. EXPRESS THE FOLLOWING VECTOR IN TERMS OF THE $\hat{i}, \hat{j}, \hat{k}$ UNIT VECTORS.

a) $(15, 11, -25)$ $\qquad 15\hat{i} + 11\hat{j} - 25\hat{k}$

b) $(4, 0, 7)$ $\qquad 4\hat{i} + 7\hat{k}$

c) $(0, -6, 29)$ $\qquad -6\hat{j} + 29\hat{k}$

d) $(0, -30, 0)$ $\qquad -30\hat{j}$

14. GIVEN: $\vec{a} = 40\hat{i} + 42\hat{j}$ AND $\vec{b} = 90\hat{i} + 56\hat{j}$,

FIND:
1) $|\vec{a}|$
2) $|\vec{b}|$
3) $\vec{a} + \vec{b}$
4) $|\vec{a} + \vec{b}|$
5) $8\vec{a}$
6) $|8\vec{a}|$
7) $\vec{a} \cdot \vec{b}$
8) $\vec{a} \times \vec{b}$
9) $|\vec{a} \times \vec{b}|$
10) θ, THE ANGLE BETWEEN $\vec{a}$ AND $\vec{b}$, BY TWO DIFFERENT METHODS.

(58, 106, (130, 98, 0), 162.8, (320, 336, 0), 464, 5952, (0, 0, −1540), 1540, 14.5°)

15. GIVEN: $\vec{a} = 48\hat{i} + 44\hat{j} + 33\hat{k}$ AND
$\vec{b} = 24\hat{i} + 18\hat{j} + 72\hat{k}$

FIND THE TEN ITEMS LISTED IN PROBLEM 14.
(73, 78, (72, 62, 105), 141.6, (384, 352, 264), 584, 4320, (2574, −2664, −192), 3709.3, 40.65°)

16. GIVEN: $\vec{a} = (54, 72, -56)$ $\quad \vec{b} = (-21, 28, 84)$, FIND $|\vec{a}|$, $|\vec{b}|$, $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$, $|\vec{a} \times \vec{b}|$ AND θ BY TWO INDEPENDENT METHODS. DRAW A TWO-DIMENSIONAL DIAGRAM OF $\vec{a}$ AND $\vec{b}$. LABEL ALL ANGLES. (106, 91, −3822, (7616, −3360, 3024), 8856.5, 113°)

CALCULUS

1. FIND THE DERIVATIVE OF Y:

A) $Y = 7X^3 + 2X + 3$

B) $Y = 5e^{4X}$

C) $Y = 4 \sin 3X$

D) $Y = 3 \cos(X/6)$

2. FIND THE MAXIMUM AND MINIMUM VALUES OF Y AND THE VALUES OF X AT WHICH THEY OCCUR.

$$Y = X^3 - 6X^2 - 15X + 7$$

3. EVALUATE THE INTEGRALS:

A) $y = \int (4x^2 + 3x + 7)\, dx$

B) $y = \int e^{x}\, dx$

C) $y = \int (2x^{-1} + 3x^{-2})\, dx$

D) $y = \int (\sin 4x)\, dx$

E) $y = \int (1/4)(\cos 6x)\, dx$

4. FIND THE AREA UNDER THE CURVE : $Y = 4X^2 + 2X$ BETWEEN $X = 1$ AND $X = 3$.

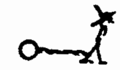
5. FIND THE AVERAGE VALUE OF $Y = 4X^2 + 2X$ BETWEEN $X = 1$ AND $X = 3$.

ANSWERS : $21X^2 + 2$, $20e^{4X}$, $12 \cos 3X$, $-\frac{1}{2} \sin(X/6)$,

MINIMUM AT $(5, -93)$ MAXIMUM AT $(-1, 15)$

$\frac{4}{3}X^3 + \frac{3}{2}X^2 + 7X + C$, $e^X + C$, $2 \ln X - 3/X + C$,

$-\frac{1}{4}(\cos 4X) + C$, $(\frac{1}{24})(\sin 6X) + C$, 42.67, 21.33

DERIVATIVES AND THE CHAIN RULE $\frac{dx}{dz} = \frac{dx}{dy}\frac{dy}{dz}$ 

1. FIND THE FOLLOWING DERIVATIVES OF THE GIVEN FUNCTIONS.

a) $\frac{dy}{dx}$ AND $\frac{dy}{dt}$ FOR $y = 8x^2$

b) $\frac{dx}{dt}$ AND $\frac{dx}{dz}$ FOR $x = 4 \sin 3t$

c) $\frac{dx}{dt}$ FOR $x = 4 \sin^5 t$ AND FOR FUNCTIONS (d) – (i).

d) $x = 2\cos^5(4t^8)$

e) $x = 2\sin^4[3(t^3+4t)^2]$

f) $x = 5e^{2t}$

g) $x = 4e^{3t^6}$

h) $x = 2e^{3(t^5+2t+1)^4}$

i) $x = e^{4\sin^3(2t^4+t)^5}$

2. GIVEN THE FUNCTIONS $x(y)$ AND $y(t)$, FIND $\dot{x}$

a) $x = 3y^4$ AND $y = 5t^3$

b) $x = \sin(3y^4)$ $y = 15t^2$

3. USING IMPLICIT DIFFERENTIATION, FIND dy/dx AS A FUNCTION OF x AND y.

a) $x^2 + y^2 = 10$

b) $y^2 - x^3 = 0$

ANSWERS: $16x$, $16x\,{}^{dx}/_{dt}$, $12\cos 3t$, $12(\cos 3t)\,{}^{dt}/_{dz}$, $20(\sin^4 t)(\cos t)$, $10(\cos^4 4t^8)(-\sin 4t^8)(32t^7)$, $8(\sin^3[3(t^3+4t)^2])(\cos[3(t^3+4t)^2])(6)(t^3+4t)(3t^2+4)$, $10e^{2t}$, $4e^{3t^6}(18t^5)$, $2\exp[3(t^5+2t+1)^4](12)(t^5+2t+1)^3(5t^4+2)$, $\exp[4\sin^3(2t^4+t)^5](12\sin^2(2t^4+t)^5)(\cos(2t^4+t)^5)(5)(2t^4+t)^4(8t^3+1)$

$12(5t^3)^3(15t^2)$, $[\cos 3(15t^2)^4](12)(15t^2)^3(30t)$

$-x/y$, $3x^2/2y$

TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

1 unified atomic mass unit,	$1\text{u} = 1.66 \times 10^{-27}\ \text{kg}$
	$= 931\ \text{MeV}/c^2$
Proton mass,	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Neutron mass,	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Electron mass,	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Magnitude of the electron charge,	$e = 1.60 \times 10^{-19}\text{C}$
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Universal gas constant,	$R = 8.31\ \text{J}/(\text{mol} \cdot \text{K})$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}\ \text{J/K}$
Speed of light,	$c = 3.00 \times 10^8\ \text{m/s}$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$
	$= 4.14 \times 10^{-15}\ \text{eV} \cdot \text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J} \cdot \text{m}$
	$= 1.24 \times 10^3\ \text{eV} \cdot \text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ \text{Wb}/(\text{A} \cdot \text{m})$
Magnetic constant,	$k' = \mu_0/4\pi = 10^{-7}\ \text{Wb}/(\text{A} \cdot \text{m})$
Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg} \cdot \text{s}^2$
Acceleration due to gravity at the Earth's surface,	$g = 9.8\ \text{m/s}^2$
1 atmosphere pressure,	$1\ \text{atm} = 1.0 \times 10^5\ \text{N/m}^2$
	$= 1.0 \times 10^5\ \text{Pa}$
1 electron volt,	$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$
1 angstrom,	$1\text{Å} = 1 \times 10^{-10}\ \text{m}$

UNITS

Name	Symbol
meter	m
kilogram	kg
second	s
ampere	A
kelvin	K
mole	mol
hertz	Hz
newton	N
pascal	Pa
joule	J
watt	W
coulomb	C
volt	V
ohm	Ω
henry	H
farad	F
weber	Wb
tesla	T
degree Celsius	°C
electron-volt	eV

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
53°	4/5	3/5	4/3
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

The following conventions are used in this examination.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Parallelepiped

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

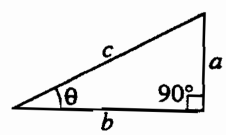
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius



CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}$$

$$\int e^x dx = e^x$$

$$\int \frac{dx}{x} = \ln |x|$$

$$\int \cos x\, dx = \sin x$$

$$\int \sin x\, dx = -\cos x$$