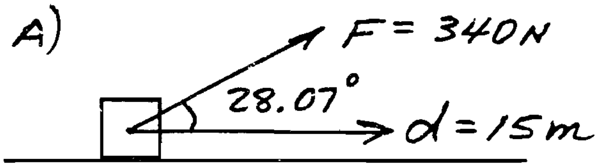
WORK

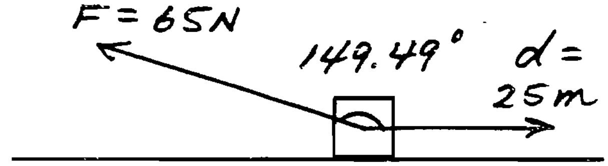
1. FIND THE WORK DONE BY THE FORCE.

A)



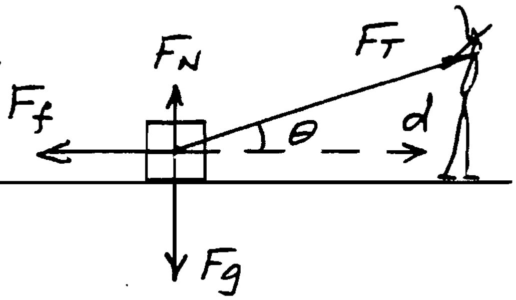
(4500 J)

B)



(−1400 J)

2.

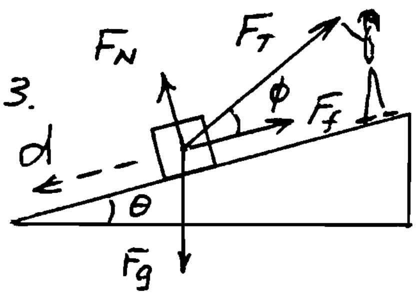


$F_T = 1460\,N$ $\quad \theta = 41.11°$
$F_N = 240\,N$ $\quad F_g = 1200\,N$
$F_f = 20\,N$ $\quad d = 50\,m$

FIND THE:

A) WORK DONE BY EACH FORCE (55000, 0, 0, −1000)
B) TOTAL WORK DONE ON THE BRICK (54000)
C) CHANGE IN KINETIC ENERGY OF THE BRICK. (54000)
D) THE INITIAL VELOCITY OF THE BRICK WAS 16 m/s. FIND ITS FINAL VELOCITY. (34 m/s)

3.



$F_T = 106\,N$ $\quad \phi = 31.89°$
$F_N = 344\,N$ $\quad F_g = 500\,N$
$\theta = 36.87°$ $\quad F_f = 110\,N$
$d = 144\,m$

FIND THE:

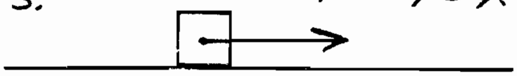
A) WORK DONE BY EACH FORCE. (−12960, 0, −15840, 43200)
B) TOTAL WORK DONE ON THE BRICK. (14400)
C) CHANGE IN KINETIC ENERGY OF THE BRICK. (14400)
D) THE INITIAL VELOCITY OF THE BRICK WAS 10 m/s. FIND ITS FINAL VELOCITY. (26 m/s)

4. FIND A FORMULA FOR THE WORK DONE BY THE FOLLOWING FORCES.

A) $F = 30x^4$
($W = 6x^5$)

B) $F = 36 \cos 4X$
($W = 9 \sin 4X$)

5. $F = 15x^2$



$m = 80$ KG $\quad v_0 = 6.78233$

$X_0 = 6 \qquad X = 10$ m

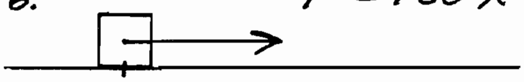
FIND THE:

A) WORK DONE BY THE FORCE FROM (6, 0) TO (10, 0).

B) CHANGING IN KINETIC ENERGY OF THE BRICK.

C) FINAL VELOCITY. (3920 J, 3920 J, 12 m/s)

6. $F = 180x^2 - 12x^3$



(0, 0)

$v_0 = 0$

A) FIND THE LOCATIONS AT WHICH THE FORCE ON THE BRICK IS ZERO. ((0,0) AND (15, 0))

B) FIND A FORMULA FOR THE WORK DONE BY THE FORCE ON THE BRICK. ($W = 60x^3 - 3x^4$)

NOTE, SINCE THE INITIAL KINETIC ENERGY IS ZERO, THIS IS ALSO A FORMULA FOR THE K.E. OF THE BRICK WHEN ITS LOCATION IS X.

C) FIND THE POINTS AT WHICH THE BRICK STOPS TO TURN AROUND. ((20, 0) AND (0, 0))

D) USE DEFINITE INTEGRALS TO FIND THE WORK FROM:

i) $X = 0$ TO $X = 12$ (41472)

ii) $X = 0$ TO $X = 15$ (50625)

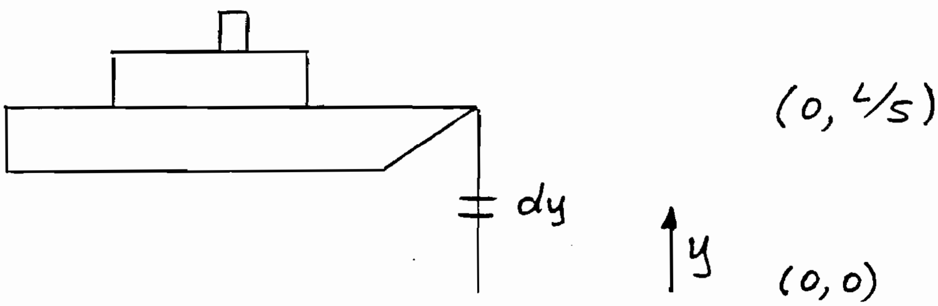
iii) $X = 12$ TO $X = 15$ (9153)

iv) $X = 15$ TO $X = 20$ (−50625)

E) THE PARTICLE WILL MAKE ONE ROUND TRIP FROM THE ORIGIN, OUT TO (20, 0), AND BACK TO THE ORIGIN, WHERE IT WILL STOP. FIND THOSE SEGMENTS DURING THE TRIP WHERE:

	ANSWERS:
i) dx IS (+)	$0 \rightarrow 20$
ii) dx IS (−)	$0 \leftarrow 20$
iii) F IS (+)	$0 \rightarrow 15$, $0 \leftarrow 15$
iv) F IS (−)	$15 \rightarrow 20$, $15 \leftarrow 20$
v) W IS (+)	$0 \rightarrow 15$, $15 \leftarrow 20$
vi) W IS (−)	$15 \rightarrow 20$, $0 \leftarrow 15$

7. A CHAIN HAS TOTAL MASS M AND TOTAL LENGTH L. ONE-FIFTH OF THE CHAIN IS HANGING OVER THE EDGE OF A SHIP. IN TERMS OF M, L AND g, FIND A FORMULA FOR THE WORK REQUIRED TO RAISE THIS SEGMENT ONTO THE DECK.



ANSWER: $MgL/50$

CONSERVATION OF ENERGY

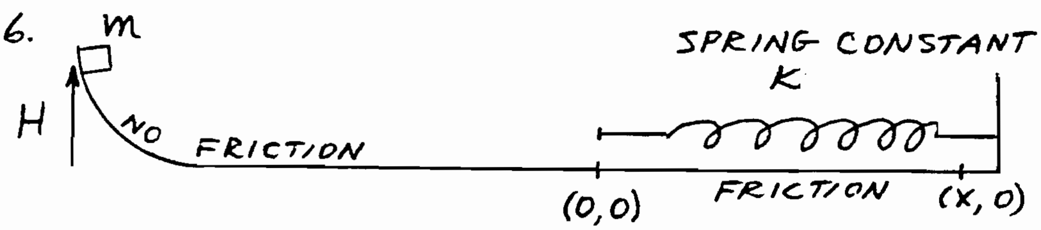
1. ATLANTA 1996! JACOB, $m = 65$ KG, WINS A GOLD MEDAL IN THE 3000 m STEEPLE CHASE, WHICH HE RUNS IN SIX MINUTES AND FIFTEEN SECONDS. FIND HIS AVERAGE KINETIC ENERGY. (2080 J)

2. TRAINING FOR THE WORLD CUP, BOBBY, $m = 60$ KG, RUNS UP 150 STAIRS IN FALCON STADIUM IN NINE MINUTES. EACH STAIR IS .3 m TALL AND .4 m WIDE. HE STARTS AND ENDS AT REST. FIND THE WORK DONE BY BOB. (27000 J)

3. ALLISON THROWS A .6 KG BASKETBALL AT 50 m/s FROM ATOP THE GYM, WHOSE ELEVATION IS 210 m. THE BALL'S PARABOLIC TRAJECTORY HAS A TOTAL CURVED LENGTH OF 300 m TO THE GROUND, WHICH IT STRIKES WITH A SPEED OF 70 m/s. FIND THE FORCE OF FRICTION ACTING ON THE BALL DURING ITS FLIGHT. (1.8 N)

4. ELENA, $m = 52$ KG, STARTS AT REST AT GROUND LEVEL. AN ELEVATOR TO THE TOP OF THE EIFFEL TOWER IN THE CITY OF LIGHTS SUPPLIES A TENSION OF 1456 N TO UNIFORMLY ACCELERATE HER UPWARD OVER A DISTANCE OF 25 m.
FIND: A) THE TOTAL WORK DONE BY THE FORCE OF TENSION.
B) THE CHANGE IN ELENA'S POTENTIAL ENERGY.
C) THE CHANGE IN HER KINETIC ENERGY.
D) HER FINAL SPEED. (36400, 13000, 23400, 30)

5. NAGANO, JAPAN 1998! STARTING FROM REST, KRISTI SKIS DOWN A HILL WHOSE ELEVATION IS 40 m. AT THE BASE OF THE HILL, SHE CROSSES THE FINISH LINE AT 16 m/s TO WIN A GOLD MEDAL. FIND THE PERCENT OF HER ORIGINAL ENERGY WHICH WAS LOST AS FRICTIONAL HEAT. (68%)

6.


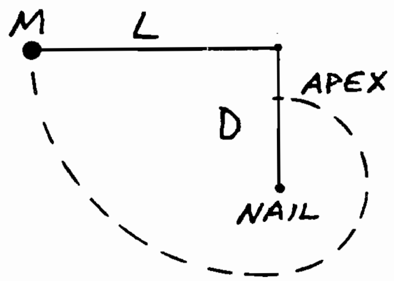
μ = COEFFICIENT OF FRICTION

THE MASS SLIDES TO THE ORIGIN AND STICKS TO THE SPRING. FIND A FORMULA FOR THE LOCATION OF THE MASS WHEN THE SPRING IS MAXIMALLY COMPRESSED. ONLY m, g, H, k AND μ ARE ALLOWED IN THE ANSWER.

ANSWER:

$$x = \left[\left(\sqrt{\mu^2 m^2 g^2 + 2kmgH}\right) - \mu mg\right] / k$$

7. A PENDULUM OF LENGTH L HAS A BOB OF MASS M. WE START THE PENDULUM HORIZONTALLY AT REST. A NAIL IS HAMMERED IN THE WALL DIRECTLY BENEATH THE FIXED END OF THE PENDULUM. THE STRING OF THE PENDULUM STRIKES THIS NAIL AND BEGIN TO WRAP AROUND IT. AT THE APEX OF THE LOOP, THE STRING'S TENSION IS N TIMES THE WEIGHT OF THE BOB. IN TERMS OF L, M, g AND N, FIND A FORMULA FOR THE DISTANCE D FROM THE FIXED END OF THE PENDULUM TO THE NAIL.



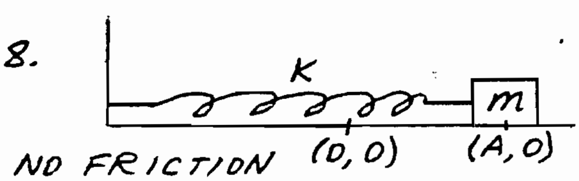
ANSWER:

$$D = \left(\frac{N+3}{N+5}\right) L$$

USE THIS ANSWER TO FIND THE MINIMUM VALUE FOR THE DISTANCE D. UNDER THOSE CONDITIONS, FIND THE TENSION IN THE ROPE WHEN THE BOB IS AT ITS APEX.

$(D = .6L \,,\; 0)$

8.



WE RELEASE THE MASS AT $(A, 0)$ FROM REST.

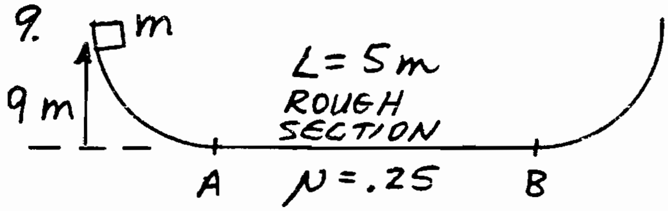
IN TERMS OF K, m AND A, FIND FORMULAS FOR:

A) THE POINT $(x, 0)$ AT WHICH THE MASS HAS ATTAINED HALF OF ITS MAXIMUM SPEED.

B) THE POINT $(x, 0)$ AT WHICH SEVENTY-FIVE PERCENT OF THE SYSTEM'S ENERGY IS POTENTIAL.

ANSWERS: $x = \dfrac{A\sqrt{3}}{2}$, $x = A\sqrt{.75}$

9.



THE UPWARD SLOPING SECTIONS OF THE TRACK HAVE NO FRICTION. THE MASS IS RELEASED FROM REST AT ELEVATION 9 m. THE HORIZONTAL ROUGH SECTION, WHOSE COEFFICIENT OF FRICTION IS .25, IS 5 m LONG. FIND THE:

A) TOTAL DISTANCE TRAVELED BY THE BRICK OVER THE ROUGH ZONE UNTIL IT COMES TO REST. (36 m)

B) WHICH DIRECTION WAS THE BRICK TRAVELING WHEN IT STOPPED? (TO THE LEFT)

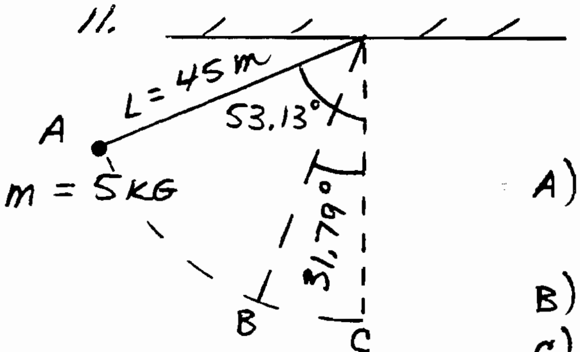
C) FIND THE DISTANCE FROM POINT A TO THE FINAL LOCATION OF THE BRICK. (4 m)

10. A 2400 KG ELEVATOR FALLS FROM REST AT AN ELEVATION OF 5 m ABOVE A SPRING WHOSE CONSTANT IS 16000 N/m.

A) FIND THE DISTANCE THROUGH WHICH THE SPRING IS COMPRESSED WHEN THE ELEVATOR FINALLY COMES TO REST. (1.5 m)

B) THE FRICTIONAL FORCE BETWEEN THE ELEVATOR AND THE SHAFT IS 9200 N. FIND THE TOTAL DISTANCE TRAVELED BY THE ELEVATOR AS ITS OSCILLATIONS DECAY AWAY AND IT COMES TO REST. (15 m)

11.



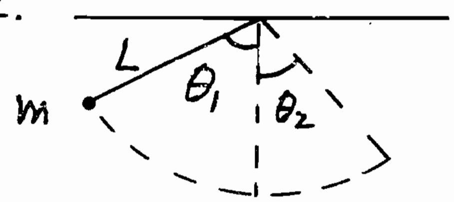
WE RELEASE THE BOB AT POINT A FROM REST. AT POINTS A, B AND C, FIND THE:

A) TANGENTIAL ACCELERATION ($8, 5.27, 0\ m/s^2$)

B) SPEED (0, 15, 19 m/s)

C) TENSION (30, 67.5, 90 N)

12.



WE RELEASE THE MASS FROM REST AT ANGLE θ_1. ON THE FIRST OSCILLATION, THE BOB ONLY ATTAINS ANGLE θ_2 AS A RESULT OF FRICTION. θ IS MEASURED IN [RADIANS].

RECALL THAT ARC LENGTH = $L\theta$.

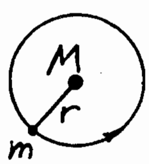
A) IN TERMS OF m, g, L, θ_1 AND θ_2, FIND A FORMULA FOR THE FORCE OF FRICTION.

B) IN TERMS OF m, g, L, θ_1 AND θ_2, FIND A FORMULA FOR THE TOTAL DISTANCE TRAVELED BY THE BOB AS IT COMES TO REST.

ANSWERS:

$$F_f = \frac{mg(\cos\theta_2 - \cos\theta_1)}{(\theta_1 + \theta_2)}, \quad D = \frac{L(1-\cos\theta_1)(\theta_1+\theta_2)}{(\cos\theta_2 - \cos\theta_1)}$$

13. NEWTON'S LAW OF GRAVITATION IS $F_g = GmM/r^2$



A SATELLITE OF MASS m ORBITS A PLANET OF MASS M AT DISTANCE r. IN THE FOLLOWING, ONLY G, m, M AND r ARE ALLOWED.

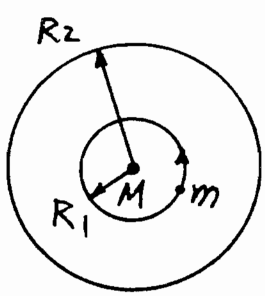
A) USE DYNAMICS TO FIND A FORMULA FOR THE KINETIC ENERGY OF THE SATELLITE.

B) USE THE DEFINITION OF POTENTIAL ENERGY TO FIND A FORMULA FOR THE P.E. OF THE SATELLITE. ASSUME THAT $U = 0$ AT $r = \infty$.

ANSWERS:

$$KE = \frac{GmM}{2r} \qquad PE = \frac{-GmM}{r}$$

14.



A SATELLITE, m, ORBITS A PLANET, M, AT RADIUS R_1. IN TERMS OF G, m, M, R_1 AND R_2, FIND A FORMULA FOR THE WORK REQUIRED TO BOOST THE ROCKET TO ORBIT OF RADIUS R_2.

ANSWER:

$$W = \frac{GmM}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

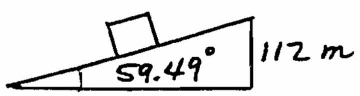
WE'VE GOT THE POWER!

1 HORSEPOWER = 746 WATTS = 550 ft·lbs/SEC

1. JOHN, $m = 72$ KG, CLIMBED FROM ELEVATION 3100 m TO THE SUMMIT OF TORREY'S PEAK AT 4300 m. THE HIKE REQUIRED 4.8 HOURS. FIND THE AVERAGE POWER SUPPLIED.
   (50 WATTS)

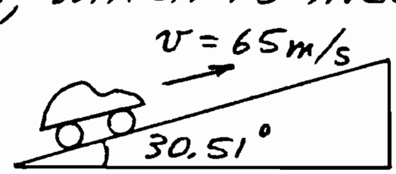
2. A WINDLASS RAISES A 200 KG ANCHOR AT 3 m/s.
   A) FIND THE AVERAGE POWER REQUIRED. (6000 WATTS)
   B) THE MOTOR IS 40% EFFICIENT. FIND THE SIZE OF THE ENGINE. (15000 WATTS)

3. IN EIGHT MINUTES, A CONVEYOR BELT, WHOSE ENGINE IS 40% EFFICIENT, LIFT 160 CRATES, EACH HAVING A MASS OF 30 KG. THE BELT, INCLINED AT 59.49°, RESULTS IN AN ELEVATION INCREASE OF 112 m. THE CRATES START AND END AT REST. FIND THE SIZE OF THE ENGINE REQUIRED TO RUN THIS BELT. (28 KWATTS)



4. THE OLYMPIC SWIMMING POOL 1996! DANNY, $m = 75$ KG, IS SWIMMING AT 2 m/s. THE END IS NEAR. IN FOUR SECONDS, HE ACCELERATES TO FOURTEEN m/s TO WIN THE GOLD MEDAL. FIND: A) THE WORK HE DID DURING THE FOUR SECOND SPRINT. B) THE AVERAGE POWER DURING THE SPRINT C) HIS ACCELERATION D) HIS VELOCITY AT THREE SECONDS INTO THE SPRINT E) HIS INSTANTANEOUS POWER AT THAT TIME.
   (7200 J, 1800 W, 3 m/s^2, 11 m/s, 2475 W)

5. PAIGE CRUISES AT A CONSTANT 65 m/s IN HER 1500 KG BMW UP MT. SOLEDAD, WHICH IS INCLINED AT 30.51°. FIND HER:
   A) v_y
   B) INSTANTANEOUS POWER



(33 m/s, 495000 WATTS)

6. AMBER'S 2500 KG BMW ACCELERATES FROM 8 m/s TO 52 m/s IN FOUR SECONDS AS IT ZOOMS UP PIKES PEAK WHOSE ELEVATION IS 72 m AND WHOSE INCLINE IS 36.9° FIND THE:

A) TOTAL WORK DONE BY THE ENGINE.

B) AVERAGE POWER FOR THE ENTIRE TRIP.

C) v_y THREE SECONDS INTO THE TRIP.

D) INSTANTANEOUS POWER THREE SECONDS AFTER BEGINNING TO ACCELERATE.

(5.1×10^6, 1.275×10^6, 24.6, 1742500)

7. DURING LIFT-OFF, THE TOTAL WORK PERFORMED BY A ROCKET ENGINE AS TIME GOES BY IS GIVEN BY: $W = 2t^3 + 8t^2$.

A) AS A FUNCTION OF TIME, FIND A FORMULA FOR THE INSTANTANEOUS POWER OF THE ENGINE. ($P = 6t^2 + 16t$)

B) FIND THE CUMULATIVE WORK PERFORMED BY THE ENGINE AT $t = 5$ SECONDS. (450 J)

C) FIND THE WORK DONE BETWEEN $t = 2$ AND $t = 5$. (402 J)

D) FIND THE AVERAGE POWER BETWEEN $t = 2$ AND $t = 5$. (134 W)

E) FIND THE INSTANTANEOUS POWER AT $t = 2$ AND AT $t = 5$ SECONDS. (56 W, 230 W)

8. ACCELERATING FROM REST, A LOCOMOTIVE ENGINE DELIVERS INSTANTANEOUS POWER GIVEN BY: $P = 12t^2$.

A) AS A FUNCTION OF TIME, FIND A FORMULA FOR THE CUMULATIVE WORK PERFORMED BY THE ENGINE. ($W = 4t^3$)

B) FIND THE INSTANTANEOUS POWER AT $t = 2$ AND AT $t = 5$. (48, 300 W)

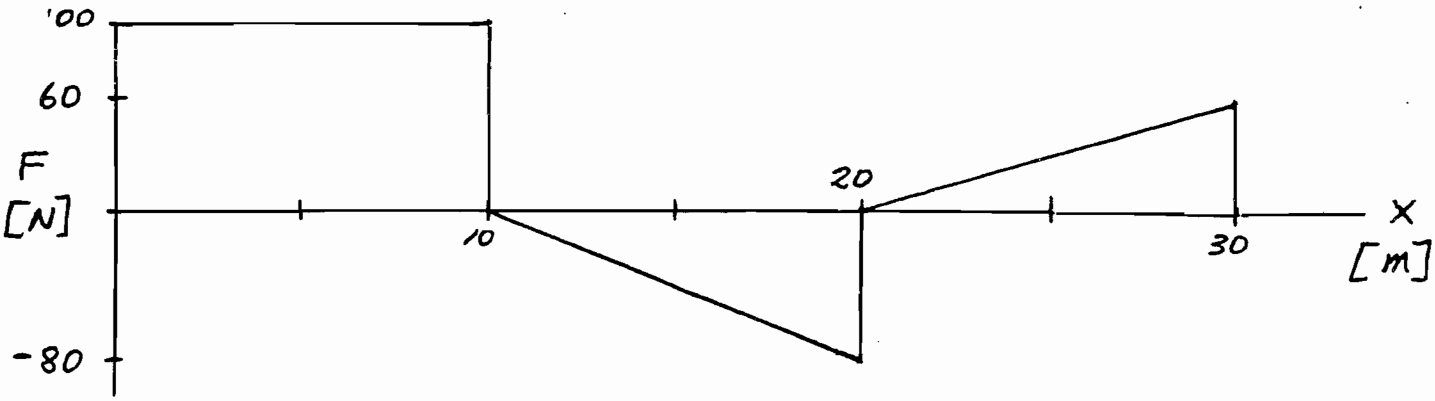
C) FIND THE CUMULATIVE WORK DONE AT $t = 5$. (500 J)

D) FIND THE WORK DONE BETWEEN $t = 2$ AND $t = 5$ (468 J)

E) FIND THE AVERAGE POWER BETWEEN $t = 2$ AND $t = 5$. (156 W)

GRAPHING

1. GIVEN :



AN OBJECT OF MASS 8 KG, INITIALLY AT REST AT THE ORIGIN, IS SUBJECTED TO A FORCE WHOSE GRAPH IS SHOWN ABOVE.

A) WITH CELERITY, FIND THE TOTAL WORK DONE BY THE FORCE.

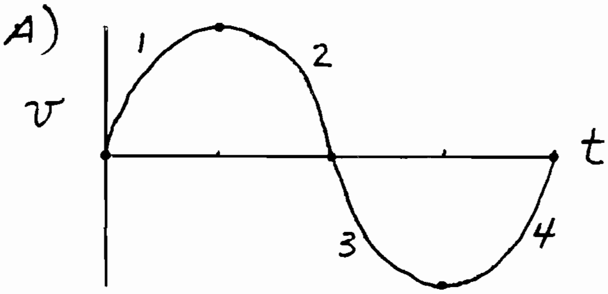
B) FIND THE FINAL SPEED OF THE PARTICLE.

C) FIND THE CHANGE IN THE KINETIC ENERGY OF THE MASS BETWEEN $X = 15$ AND $X = 25\ m$.

(900 J, 15 m/s, −225 J)

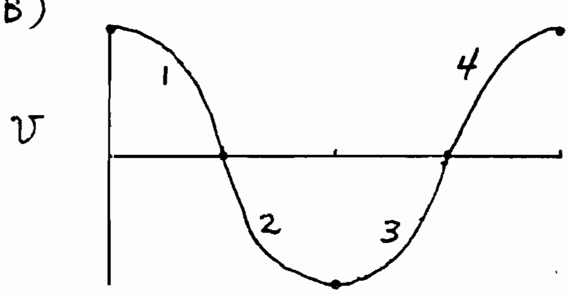
2. FOR EACH CURVE, FIND THE PARITY OF THE ACCELERATION, THE VELOCITY AND dx, AND THE WORK DONE DURING THAT SECTION OF THE TRIP.

A)



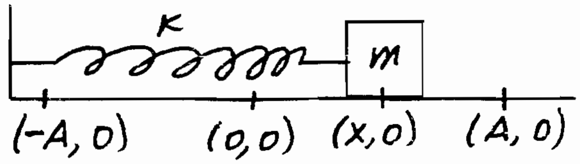
	a	v dx	W
1.			
2.			
3.			
4.			

B)



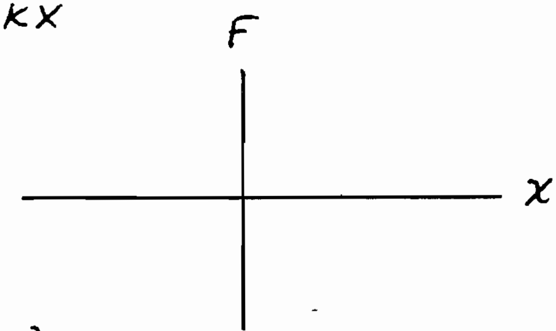
	a	v dx	W
1.			
2.			
3.			
4.			

3. A MASS OSCILLATES BETWEEN $(A,0)$ AND $(-A,0)$ ON THE END OF A SPRING WHICH FOLLOWS HOOKE'S LAW.

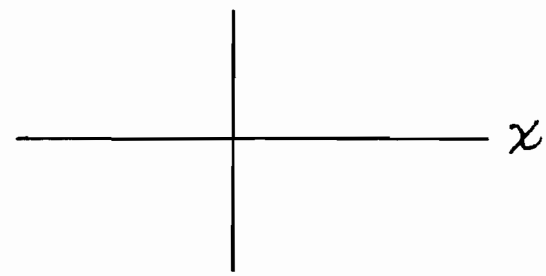


SKETCH GRAPHS BELOW. LABEL THE AXES.

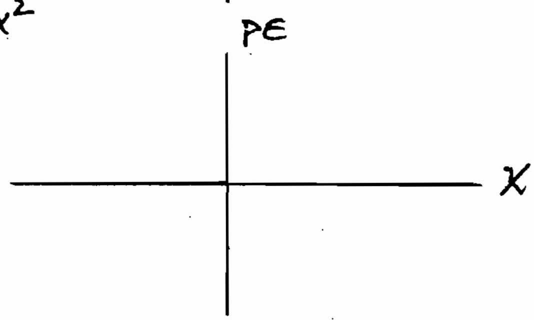
$F = -KX$

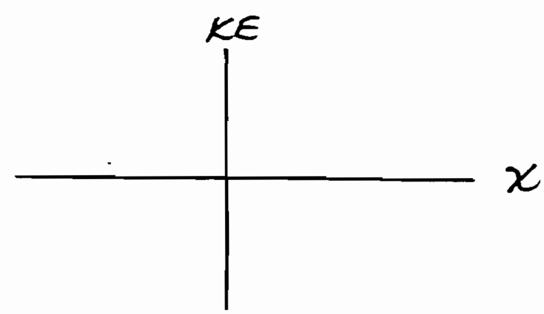


TOTAL ENERGY

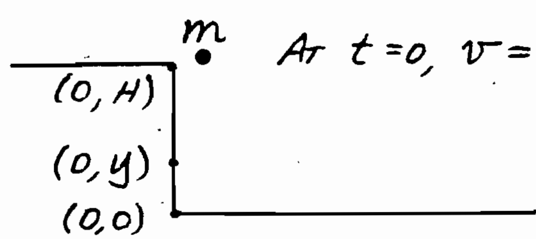


$PE = \frac{1}{2}KX^2$

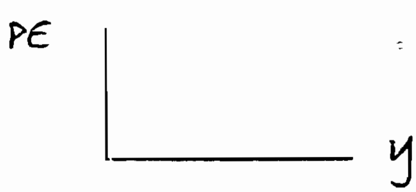


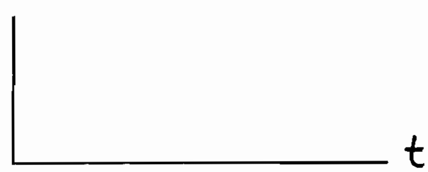


4. A PARTICLE IS RELEASED FROM REST AND FALLS TO THE GROUND. SKETCH THE GRAPHS BELOW, LABELING THE AXES.



AT $t=0$, $v=0$.





MULTI-DIMENSIONAL CONSERVATIVE SYSTEMS

1. THE POTENTIAL ENERGY IS GIVEN BY:

$$\mathcal{U} = 3x^2 + 2xy + 6y^2 - 18x + 28y$$

A. FIND AN EXPRESSION FOR THE FORCE ASSOCIATED WITH THIS POTENTIAL ENERGY.

B. CALCULATE THE VALUE OF THE POTENTIAL ENERGY AT $(0,0)$.

C. CALCULATE THE CHANGE IN THE POTENTIAL ENERGY AS WE MOVE FROM LOCATION $(1,2)$ TO $(5,3)$.

D. FIND THE EQUILIBRIUM POINTS, THAT IS, THE POINTS AT WHICH $\vec{F} = (0,0)$.

E. IS THIS EQUILIBRIUM POINT STABLE, UNSTABLE OR A SADDLE POINT?

F. CALCULATE THE POTENTIAL ENERGY AT THE EQUILIBRIUM POINT.

2. A FORCE IS GIVEN BY THE FORMULA:

$$\vec{F} = -\hat{\imath}\,(12x + 6y + 18) - \hat{\jmath}\,(6x - 2y - 16)$$

A. FIND A FORMULA FOR THE POTENTIAL ENERGY ASSOCIATED WITH THIS FORCE. ASSUME THAT $\mathcal{U} = 0$ AT THE ORIGIN $(0,0)$.

B. FIND THE EQUILIBRIUM POINT. IS IT STABLE, UNSTABLE OR A SADDLE POINT?

ANSWERS: $\vec{F} = -\hat{\imath}\,(6x + 2y - 18) - \hat{\jmath}\,(2x + 12y + 28)$

0, $84\,J$, $(4,-3)$, STABLE, $-78\,J$, $\mathcal{U} = 6x^2 + 6xy + 18x - y^2 - 16y$

$(1,-5)$, SADDLE: STABLE ON X, UNSTABLE ON Y.

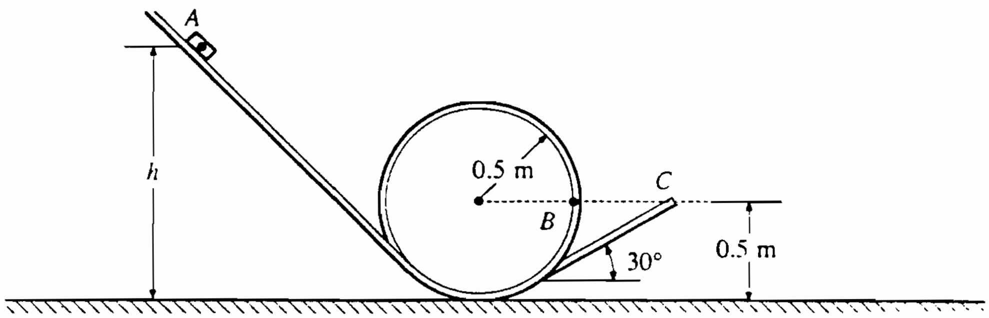
PHYSICS C

SECTION II. MECHANICS

Time—45 minutes

3 Questions

ANSWER ALL OF THE QUESTIONS. EACH OF THE THREE QUESTIONS HAS EQUAL WEIGHT. BUT THE PARTS WITHIN A QUESTION MAY NOT HAVE EQUAL WEIGHT. SHOW YOUR WORK. CREDIT FOR YOUR ANSWERS DEPENDS ON THE QUALITY OF YOUR EXPLANATIONS.



1. A 0.1-kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 meters per second at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

(a) Determine the height h. (1.3 m)

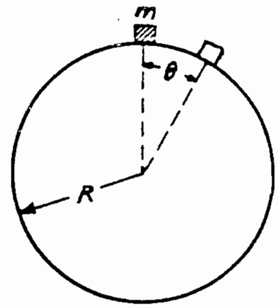
(b) On the figure below, draw and label all the forces acting on the block when it is at point B, which is 0.5 meter above the ground.

(c) Determine the magnitude of the force exerted by the track on the block when it is at point B. (3.2 N)

(d) Determine the maximum height above the ground attained by the block after it leaves the track. (.7 m)

(e) Another track that has the same configuration, but is NOT frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 meters per second, the height h must be 2 meters. Determine the work done by the frictional force. (.7 J)

$mgR(1-\cos\theta)$
$2g(1-\cos\theta)$
$g\sin\theta$
$\cos^{-1}(2/3)$



2. A particle of mass m slides down a fixed, frictionless sphere of radius R, starting from rest at the top.

(a) In terms of m, g, R, and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass

iii. The tangential acceleration of the mass

(b) Determine the value of θ at which the particle leaves the sphere.

3. A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the cube of the displacement; i.e.,

$$F = -kx^3.$$

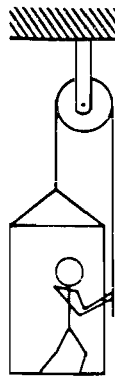
This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

(a) The potential energy in the spring at the instant the mass is released $U = kA^4/4$

(b) The maximum speed of the mass $[kA^4/2m]^{1/2}$

(c) The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

$$\frac{A}{\sqrt[4]{2}}$$



4. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.

(a) If the platform and the person are at rest, what is the tension in the rope? (500 N)

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

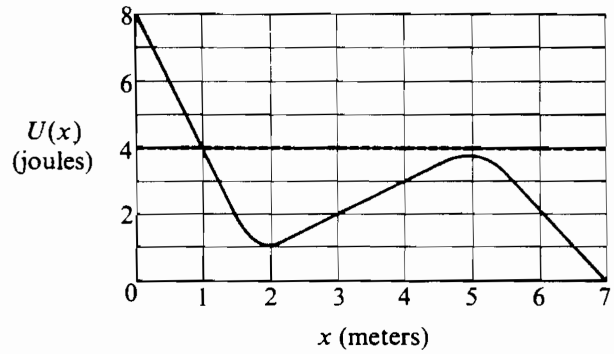
(b) What is the tension in the rope under these new conditions? (600 N)

(c) Under these conditions, what is the force exerted by the platform on the person? (360 N)

After a short time, the person and the platform reach and sustain an upward velocity of 0.4 m/s.

(d) Determine the power output of the person required to sustain this velocity. (400 WATTS)

5. The following graph shows the potential energy $U(x)$ of a particle as a function of its position x.



(a) Identify all points of equilibrium for this particle.

Suppose the particle has a constant total energy of 4.0 joules, as shown by the dashed line on the graph.

(b) Determine the kinetic energy of the particle at the following positions

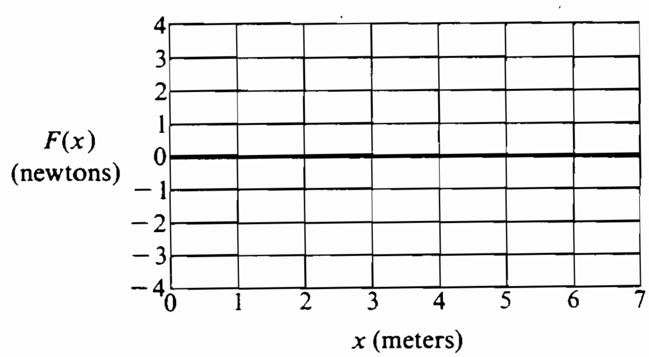
i. $x = 2.0$ m

ii. $x = 4.0$ m

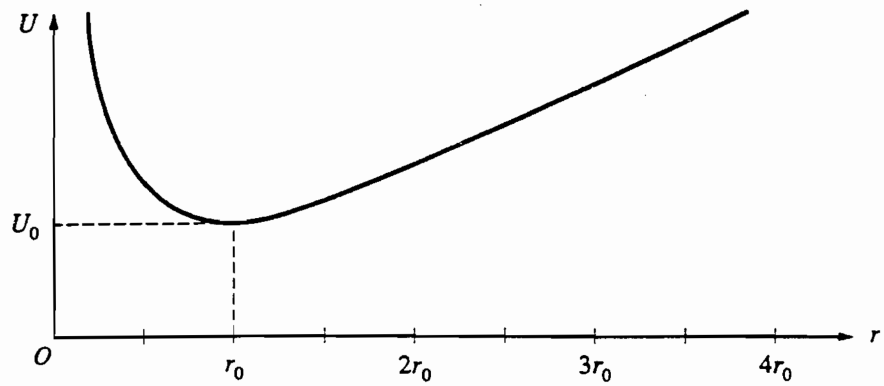
(c) Can the particle reach the position $x = 0.5$ m ? Explain.

(d) Can the particle reach the position $x = 5.0$ m ? Explain.

(e) On the grid below, carefully draw a graph of the conservative force acting on the particle as a function of x, for $0 < x < 7$ meters.



6. A particle of mass m moves in a conservative force field described by the potential energy function $U(r) = a(r/b + b/r)$, where a and b are positive constants and r is the distance from the origin. The graph of $U(r)$ has the following shape.

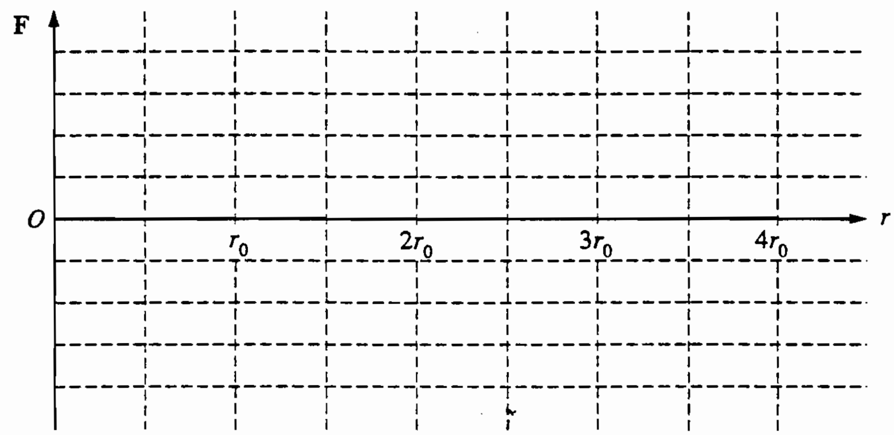


(a) In terms of the constants a and b, determine the following.

i. The position r_0 at which the potential energy is a minimum $(r_0 = b)$

ii. The minimum potential energy U_0 $(U_0 = 2a)$

(b) Sketch the net force on the particle as a function of r on the graph below, considering a force directed away from the origin to be positive, and a force directed toward the origin to be negative.



The particle is released from rest at $r = r_0/2$.

(c) In terms of U_0 and m, determine the speed of the particle when it is at $r = r_0$.

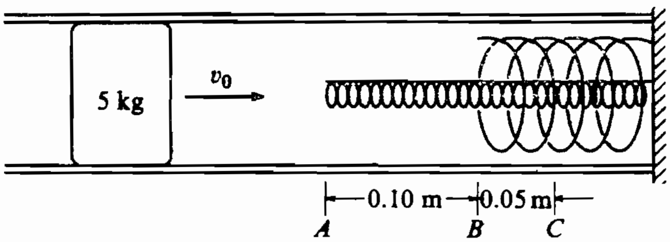
(d) Write the equation or equations that could be used to determine where, if ever, the particle will again come to rest. It is not necessary to solve for this position.

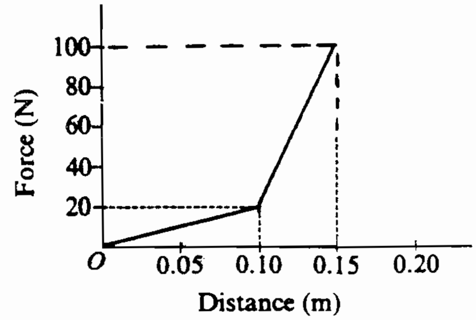
(e) Briefly and qualitatively describe the motion of the particle over a long period of time.

ANSWERS: c) $v = \sqrt{\frac{U_0}{2m}}$ d) $\frac{5}{2} = \frac{r}{b} + \frac{b}{r}$

e) IT OSCILLATES BACK AND FORTH BETWEEN $r_0/2$ AND ABOUT $2r_0$. IT ACCELERATES AND DECELERATES QUICKEST TO THE LEFT OF r_0.

M M M M M M M M M M M M M





7. A 5-kilogram object initially slides with speed v_0 in a hollow frictionless pipe. The end of the pipe contains two springs, one nested inside the other, as shown above. The object makes contact with the inner spring at point A, moves 0.1 meter to make contact with the outer spring at point B, and then moves an additional 0.05 meter before coming to rest at point C. The graph shows the magnitude of the force exerted on the object by the springs as a function of the object's distance from point A.

(a) Calculate the spring constant for the inner spring.

(b) Calculate the decrease in kinetic energy of the object as it moves from point A to point B.

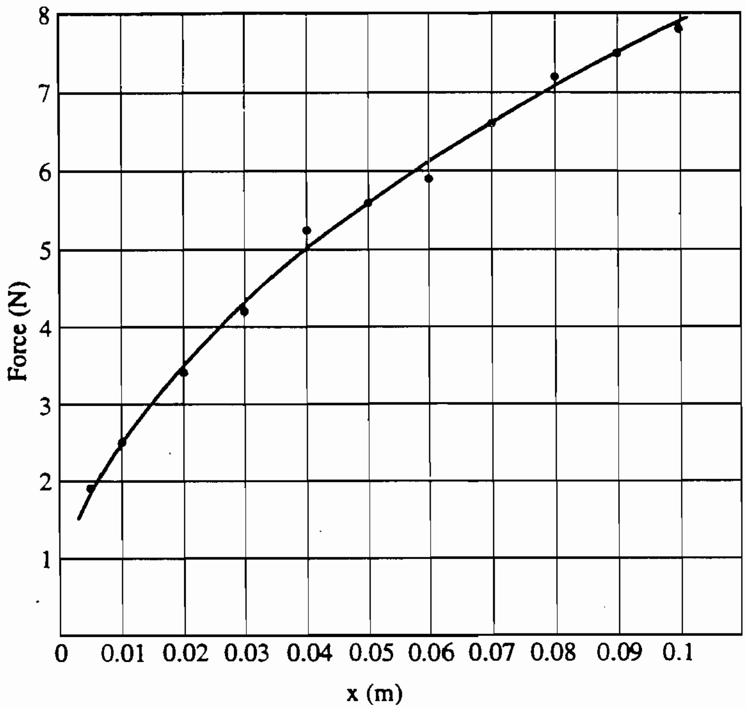
(c) Calculate the additional decrease in kinetic energy of the object as it moves from point B to point C.

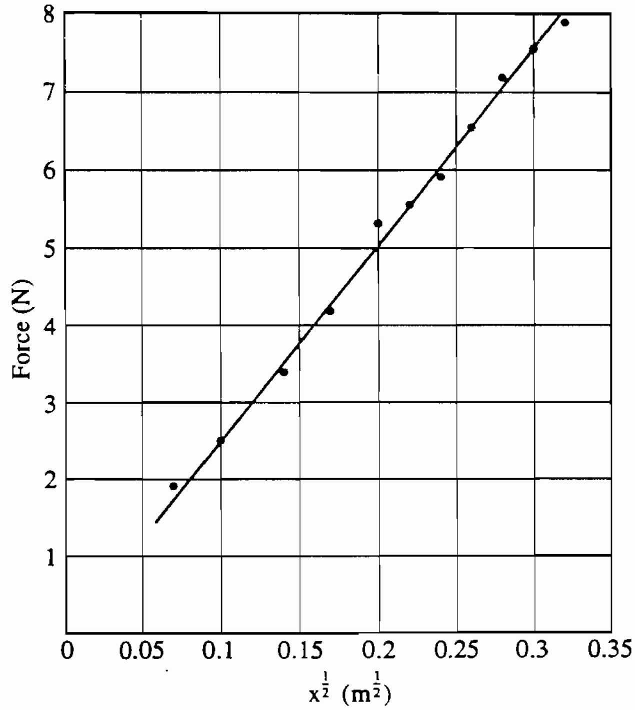
(d) Calculate the initial speed v_0 of the object.

(e) Calculate the spring constant of the outer spring.

200 N/m, 1 J, 3 J, $\sqrt{8/5}$ m/s, 1400 N/m

8. A nonlinear spring is compressed horizontally. The spring exerts a force that obeys the equation $F(x) = Ax^{1/2}$, where x is the distance from equilibrium that the spring is compressed and A is a constant. A physics student records data on the force exerted by the spring as it is compressed and plots the two graphs below, which include the data and the student's best-fit curves.





(a) From one or both of the given graphs, determine A. Be sure to show your work and specify the units.

(b) i. Determine an expression for the work done in compressing the spring a distance x.

ii. Explain in a few sentences how you could use one or both of the graphs to estimate a numerical answer to part (b) i for a given value of x.

(c) The spring is mounted horizontally on a countertop that is 1.3 m high so that its equilibrium position is just at the edge of the countertop. The spring is compressed so that it stores 0.2 J of energy and is then used to launch a ball of mass 0.10 kg horizontally from the countertop. Neglecting friction, determine the horizontal distance d from the edge of the countertop to the point where the ball strikes the floor.

ANSWERS:

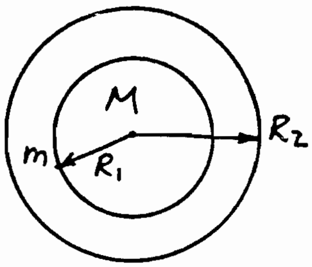
a) $A = 25\ [N/m^{1/2}]$ $\therefore\ F = 25X^{1/2}$

b) $W = \frac{50}{3} X^{3/2}$ FIND AREA UNDER EXTRAPOLATED F − X CURVE. COUNT BOXES AND TRIANGLES.

c) 1 m

NEWTON'S LAW OF GRAVITATION: $F_g = GmM/R^2$

9. A SATELLITE OF MASS m IS INITIALLY ORBITING AT RADIUS R_1 FROM THE CENTER OF THE PLANET OF MASS M.



A) FIND A FORMULA FOR THE INITIAL K.E. OF THE SATELLITE. ONLY G, m, M AND R_1 MAY BE IN YOUR ANSWER.

$KE =$ ____________________

B) STARTING FROM THE DEFINITION OF POTENTIAL ENERGY, FIND A FORMULA FOR THE INITIAL P.E. OF THE SATELLITE. ONLY G, m, M AND R_1 ARE ALLOWED. SHOW ALL WORK. ASSUME THE U AT $r = \infty$ IS ZERO.

$$U_{R_1} - U_\infty = U_{R_1} = -\int \vec{F} \cdot d\vec{l}$$

$U_{R_1} =$ ____________________

C) FIND A FORMULA FOR THE WORK REQUIRED TO MOVE THE SATELLITE FROM ORBIT R_1 TO R_2. ONLY G, m, M, R_1 AND R_2 ...

$W =$ ____________________